OPTIMAL CAPACITOR PLACEMENT ON RADIAL DISTRIBUTION FEEDERS IN PRESENCE OF NONLINEAR LOADS USING BINARY PARTICLE SWARM OPTIMIZATION

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ABSTRACT

Harmonic distortion is increasing in distribution power systems due to the proliferation of nonlinear distorting loads. Capacitor significantly influences the propagation of system harmonics, and could cause parallel resonance. Therefore, the optimal selection and placement of capacitor banks must be integrated with the estimation of harmonic levels to avoid excessive harmonic distortion. The problem of capacitor placement involves maximizing of the net saving with inequality constraints. This paper proposes a binary particle swarm optimization (PSO) to solve the capacitor placement problems. The proposed method has been applied to: (1) IEEE 9-bus test system and the results are compared with that obtained using exhaustive search. (2) IEEE 34-bus test system. The simulation results show the ability of the binary PSO to solve the capacitor placement problems and investigate the effect of nonlinear loads on optimal capacitor placement.

INTRODUCTION

Reactive currents in distribution systems produce losses and result in increased ratings for distribution components. The losses produced by reactive currents can be reduced by the installation of shunt capacitors. Capacitors are widely installed in distribution systems for reactive power compensation to achieve power and energy loss reduction, power factor correction, system capacity release and to maintain a voltage profile within permissible limits.

The general capacitor placement problem attempts to determine the (i) location, (ii) types (switched or fixed), (ii) size of capacitors to be allocated in the nodes of a radial distribution system at various loading conditions such that the economic benefits due to peak power and energy loss reduction be weighted against the cost of installment of such capacitors while the operational constraints (e.g. the voltage profile) at different load levels are satisfied [1].

Most of the capacitor placement techniques assume that all loads are linear and ignore the effect of harmonics. Limited publications have taken into consideration the presence of harmonics when solving the capacitor placement problems [2].

Particle swarm optimization (PSO), first introduced by Kennedy and Eberhart [3], is one of the modern heuristic algorithms. It was developed through simulation of a simplified social system, and has been found to be robust in solving continuous nonlinear optimization problems. Kennedy and Eberhart [4] also developed the discrete binary version of the PSO.

In this work, we propose the binary PSO algorithm to solve capacitor placement problems in the presence of nonlinear loads. The binary PSO algorithm applied to two test systems. The first one is IEEE 9-bus test system given in [5] with the same formulation, assumption and constraints in the presence of the harmonics which injected by nonlinear loads. The numerical example shows the success of the binary PSO in solving capacitor placement problems. The second is IEEE 34-bus test system with nonlinear loads used to demonstrate the effect of nonlinear loads on optimal capacitor placement.

SYSTEM MODEL AT FUNDAMENTAL AND HARMONIC FREQUENCIES

Figure 1 shows an *m*-bus radial distribution feeder and a forecasted load duration curve which is approximated by *L* discrete load levels. For modelling of a distribution system shown in Figure (1.a) at fundamental and harmonic frequencies the formulation and notations and assumption of [5] are used, where all loads vary in a conforming way, i.e. the *l-th* load level at the *i*-th bus can be expressed in terms of peak load by $(P_{il}, Q_{il}) = x_l\,(P_{i1}, Q_{i1})$ where $x_l \leq 1$.

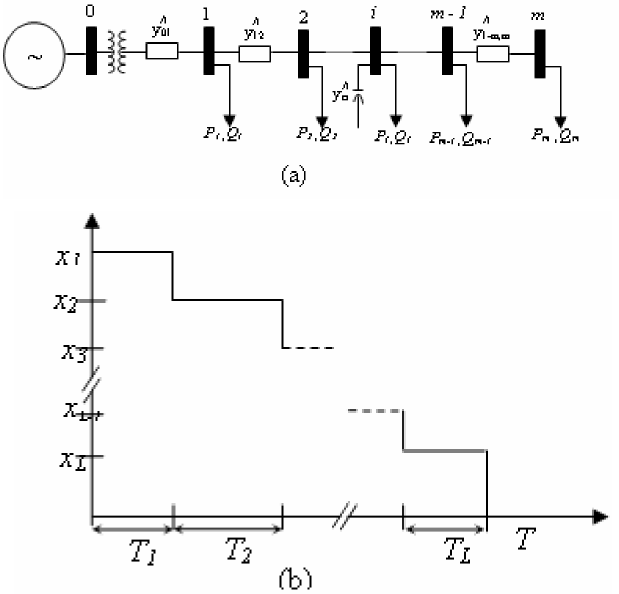

Figure 1: (a) One-line Diagram of a Radial Distribution Feeder (b) Discretized Load Duration Curve

The modelling steps at *l-th* load level are as follows:

- Calculate the magnitudes and the phase angles of the bus voltages at fundamental frequency by using Newton-Raphson power flow method
- Calculate the fundamental current, *h-th* harmonic current, *h-th* harmonic frequency load admittances, shunt capacitor admittances and feeder admittances using Eqns. (1)-(5) respectively.
- Calculate the harmonic voltages caused by nonlinear loads from Eqn. (6)
- Calculate the rms voltage and the total harmonic distortion using Eqns. (7)-(8) respectively.

$$I_{il}^{1} = (P_{il} - jQ_{il}) \big/ (V_{il}^{1})^{*} \qquad (1)$$

$$I_{il}^{h} = w_i I_{il}^{1} \big/ h \qquad (2)$$

$$y_{il}^{load,h} = (1 - w_i)(P_{il} - jQ_{il}/h) \big/ \left|V_{il}^{1}\right|^{2} \qquad (3)$$

$$y_c^h = h y_c^1 \qquad (4)$$

$$y_{i,i+1}^{h} = \left(R_{i,i+1} + jhX_{i,i+1}\right)^{-1} \qquad (5)$$

$$\begin{bmatrix} V_{1l}^h \\ V_{2l}^h \\ . \\ . \\ . \\ V_{ml}^h \end{bmatrix} = \begin{bmatrix} Y_{11}^{lh} & Y_{11}^{lh} & 0 & & & 0 \\ Y_{11}^{lh} & Y_{11}^{lh} & . & & & \\ 0 & . & . & & & \\ . & & & . & . & 0 \\ . & & & . & . & Y_{m-1,m}^{lh} \\ 0 & & & 0 & Y_{m,m-1}^{lh} & Y_{m,m}^{lh} \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_{1l}^h \\ I_{2l}^h \\ . \\ . \\ . \\ I_{ml}^h \end{bmatrix} \qquad (6)$$

$$Y_{ij}^{lh} = \begin{cases} -y_{ij}^h & if \quad j \neq i \\ y_{i-1,i}^h + y_{i+1,i}^h + y_{il}^{load,h} + (u_{fj}^l + u_{sj}^l) y_c^h & if \quad j = i \in S_c \end{cases}$$

$$|V_{il}| = \sqrt{\sum_{h=1}^{H} |V_{il}^h|^2} \qquad (7)$$

$$THD_{il}(\%) = \frac{100}{|V_{il}^1|} \sqrt{\sum_{h \neq 1}^{H} |V_{il}^h|^2} \qquad (8)$$

Where

w_i	the nonlinear portion of the load at bus *i*.
$(V_{il}^1)^*$	the complex conjugate of the fundamental voltage at *l-th* load level and *i-th* bus.
P_{il} , Q_{il}	load active and reactive powers at *l-th* load level and *i-th* bus.
$R_{i,i+1}, X_{i,i+1}$	resistance and inductive reactance of feeder section between buses *i* and *i+1*.
Sc	the set of candidate buses for capacitor placement
u_{fj}^l, u_{sj}^l	the number of fixed and switched capacitors which placed at *l-th* load level and *j-th* bus.
H	the upper limit of considered harmonic order

PROBLEM FORMULATION

The objective function is to find the number of fixed and switched capacitors such that the net savings resulting from peak power and total energy loss reduction, while taking capacitor cost into account, is maximized [5] with constraints that include limits on rms voltage, *THD* and number of installed capacitors.

- The net savings is computed by Eqn. (9), where the total peak losses *(l=1)*, the total energy losses and the annual shunt capacitor cost are given by Eqns. (10)-(12) respectively.
- It is assumed that the selection of capacitors is limited to one standard size *Qc* (e.g., 300 kvar), where *u* and *U* denote the maximum number of capacitors allowed at each location and on the entire feeder, respectively.
- It desired to satisfy the constraints in Eqns. (13)-(16)

$$S = K^p (\Delta P)_{loss,1} + \sum_{l=1}^{L} K^e (\Delta E)_{loss,l} - K^c \qquad (9)$$

$$P_{loss,1} = \sum_{h=1}^{H} \sum_{i=0}^{m-1} \left\{ R_{i,i+1} [\, | V_{i+1,1}^h - V_{i,1}^h \| y_{i,i+1}^h | \,]^2 \right\} \qquad (10)$$

$$E_{loss,l} = T_l P_{loss,l} \qquad (11)$$

$$K^c = \sum_{j \in S_c} (k_f^c u_{fj}^l + k_s^c u_{sj}^*) \qquad (12)$$

$$u_{sj}^* = \max\left\{ u_{sj}^l, l = L-1, L-2, \ldots\ldots, 2, 1 \right\}$$

$$u_{fj}^L + u_{sj}^l \leq u \qquad (13)$$

$$\sum_{j \in S_c} (u_{fj}^L + u_{sj}^l) \leq U \qquad (14)$$

$$V_{\min} \leq |V_{il}| \leq V_{\max} \qquad (15)$$

$$THD_{il} \leq THD_{\max} \qquad (16)$$

Where:

$(\Delta P)_{loss,1}, (\Delta E)_{loss,l}$	the difference between peak power and energy losses before and after capacitor placement.
K^p, K^e	respective constants to convert power and energy into dollars.
T_l	the load duration at *l-th* level.
$V_{\min}, V_{\max}$	Minimum and maximum permissible rms voltage
$THD_{\max}$	Maximum permissible total harmonic distortion

PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization is an algorithm developed by Kennedy and Eberhart [3] that simulates the social behaviors of bird flocking or fish schooling and the methods by which they find roosting places, foods sources or other suitable habitat.

In the basic PSO technique, suppose that the search space is d-dimensional,

- Each member is called *particle*, and each particle (*i-th* particle) is represented by *d*-dimensional vector and described as $X_i = [x_{i1}, x_{i2}, \ldots\ldots, x_{id}]$.
- The set of *n* particle in the swarm are called *population* and described as $pop = [X_1, X_2, \ldots, X_n]$.
- The best previous position for each particle is called *particle best* and described as $PB_i = [pb_{i1}, pb_{i2} \ldots, pb_{id}]$
- The best position among all of the particle best position achieved so far is called *global best* and described as $GB = [gb_1, gb_2, \ldots\ldots, gb_d]$.
- The rate of position change for each particle is called

particle velocity and described as $V_i = [v_{i1}, v_{i2}, ..., v_{id}]$.

- At iteration k the velocity for d-dimension of i-particle is updated by:

$$v_{id}^{k+1} = w v_{id}^{k} + c_1 r_1 (pb_{id}^{k} - x_{id}^{k}) + c_2 r_2 (gb_{d}^{k} - x_{id}^{k}) \quad (17)$$

  Where w is the inertia weight, c_1 and c_2 are the acceleration constants, and r_1 and r_2 are two random values in range [0,1].
- The i-particle position is updated by

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \quad (18)$$

For binary discrete search space, Kennedy and Eberhart [4] have adapted the PSO to search in binary spaces, by applying a sigmoid transformation to the velocity component given in Eqn. (19) to squash the velocities into a range [0,1], and force the component values of the locations of particles to be 0's or 1's. The equation for updating positions in Eqn. (18) is then replaced by Eqn. (20).

$$sigmoid(v_{id}^{k}) = \frac{1}{1+e^{-v_{id}^{k}}} \quad (19)$$

$$x_{id}^{k} = \begin{cases} 1, & if \;\; rand < sigmoid\left(v_{id}^{k}\right) \\ 0, & otherwise \end{cases} \quad (20)$$

FORMULATION OF CAPACITOR PLACEMENT USING BINARY PSO

Figure (1.a) shows an m-bus radial distribution feeder and Figure (1.b) shows a discretized L load level duration curve. To select the number of capacitors to be placed at *l-th* load level at j-bus (Nc_j^l), use a combination of integer multiple to reach the maximum number of capacitors allowed at each location (u) from Eqn. (21)

$$Nc_j^l = 2^0 b_1 + 2^1 b_2 + ... + 2^{r-1} b_r + ... + 2^{R-1} b_R \quad (21)$$

Where:

$2^0 + 2^1 + 2^2 + ... + 2^{r-1} + ... + 2^{R-1} \geq u$, and $b_r = \{0,1\}$

<u>For optimal capacitor placement a binary PSO will be used as follows:</u>

At l-level:

- A population of n particles at iteration k is represented by: $pop^{lk} = [X^{lk}_1, X^{lk}_2, ... , X^{lk}_i, ..., X^{lk}_n]$.
- Each particle i represented in J-dimensional (J represents the candidate buses) by:
  $X^{lk}_i = [x^{lk}_{i1}, x^{lk}_{i2}, ..., x^{lk}_{ij}, ..., x^{lk}_{iJ}]$.
- Each dimension j represented in R-dimensional (R represents the maximum integer used in Eqn. (21)) by:
  $x^{lk}_{ij} = [x^{lk}_{ij1}, x^{lk}_{ij2}, ... x^{lk}_{ijr}, ... x^{lk}_{ijR}]$.
- Therefore, each particle i is represented in *(J,R)* dimensions.
- The number of capacitors placed at load level l at bus j at iteration k in particle i is represented by :

$$Nc_{ij}^{lk} = 2^0 x^{lk}_{ij1} + 2^1 x^{lk}_{ij2} + ... + 2^{r-1} x^{lk}_{ijr} + ... + 2^{R-1} x^{lk}_{ijR}$$

- The dimension x^{lk}_{ijr} indicates if the value 2^{r-1} is used at load level l bus j at iteration k in particle i or not. In other words, x^{k}_{ijr} is a binary value such that $x^{lk}_{ijr} = 1$ if value 2^{r-1} is used at load level l bus j at iteration k in particle i, $x^{lk}_{ijr} = 0$ if it is not used.
- The particle best, global best and the particle velocity are represented also in (J, R) dimensions.

NUMERICAL EXAMPLE

The proposed binary PSO was applied to the test system described in [5] and the results were compared to those obtained in [5] using an exhaustive search. The peak load, the feeder data, the load duration and supply voltage data are presented in [5]. The radial distribution feeder has nine load busses ($m=9$) and its nominal substation transformer is rated at *15 MVA*, *23 kV* with an impedance of *(0.5+j5)%*. Values of other parameters, namely, w_i, S_c, K^p, K^e, k^c_f, k^c_s, Qc, u, U, N, V_{min}, V_{max} *and* THD_{max} are set to be equal to *15%, {3, 4, 5, 7, 9}, $120/kW, $0.05/kWh, $100/year, $100/year, 300 kvar, 10, 28, 13, 0.95 pu, 1.05 pu and 5 %* respectively.

<u>The simulation results in Table 1 show that:</u>

- Before capacitor placement: the maximum *THD* at each load level within the permissible limits, the minimum rms voltage is *0.872* and the annual total cost of the system losses is *$ 252873.5*.
- After optimal capacitor placement without voltage constraints: applying the exhaustive search shows a net saving of *13.77%*, where applying the binary PSO shows a net saving of *13.81%*.
- After optimal capacitor placement with voltage constraints: applying the exhaustive search shows a net saving of *13.45%*, where applying the binary PSO shows a net saving of *13.49%*.

TABLE 1
Optimal Number of 300 kvar Capacitors for Different Load Levels, Using Exhaustive Search (ES) and Binary Particle Swarm Optimization (PSO), (# Fixed- and * Switched-Type Capacitors)

	Load Level l	j- without voltage constraints					j- with voltage constraints				
		3	4	5	7	9	3	4	5	7	9
ES	1*	7	5	2	0	1	5	4	2	2	1
	2*	3	3	1	0	1	3	3	1	1	0
	3#	1	2	1	1	0	1	2	1	0	1
PSO	1*	6	5	2	0	1	5	4	2	1	2
	2*	4	2	0	0	1	3	2	1	0	1
	3#	2	2	1	1	0	2	2	1	1	0

The above results show the ability of the binary PSO to solve the capacitor placement problems.

If we assume that 100% of the load is linear ($w_i = 0$) and solve the previous example as mentioned in [5], there will be a considered difference among the number of capacitors at a various location , that's investigate the effect of nonlinear loads on optimal capacitor placement.

ANOTHER TEST SYSTEM

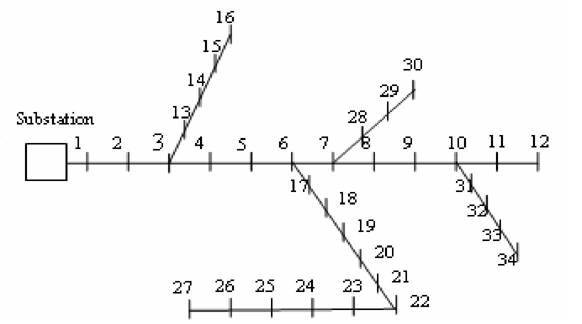

Figure 2: One-line Diagram of IEEE 34-Bus Radial Distribution Feeder

For the sake of conclusions support, the method discussed before is applied to another IEEE 34-bus radial distribution test system. A single line diagram is shown in Figure 2. The data of this feeder is presented in [6]. The system line voltage is *11 kV*. Assuming that there is only one load level, all the capacitors are fixed and Eqn. (9) doesn't include the reduction of energy losses. This test system demonstrates the effect of harmonics on the optimal capacitor placement with two cases;

(1) The harmonic effects are ignored.
(2) The harmonic effects are considered.

Values of other parameters, namely, w_i, S_c, K^p, k^c_f, Q^c, u, U, N, V_{min}, V_{max} and THD_{max} are set to be equal to *30%, {5, 10, 21, 26}, $120/kW, $100/year, 300 kvar, 10, 30, 13, 0.95 pu, 1.05 pu and 5 %* respectively.

TABLE 2

Optimal Solution for Different Cases

		Number of 300 kvar capacitor banks		
		BCP	Case 1	Case 2
Bus No.	5	0	3	10
	10	0	6	7
	21	0	6	7
	26	0	3	5
Max. voltage		1.02	1	1
Min. voltage		0.964	0.953	0.955
Max. THD		22.57	7.92	4.98
Peak power losses		221.7	160.4	171.58
Capacitor Cost($)		0	1800	2900
Power cost ($)		26606.8	19249	20590
Total cost ($)		26606.8	21049.3	23490
Benefit ($)		----	5557.5	3116.8

The simulation results in Table 2 show that:

- *Before capacitor placement:* the system is badly distorted by nonlinear loads. The resulting THDs at all buses are over (21%). The annual total cost of the system losses is $ 26606.8.
- *After capacitor placement*:

Case 1) the harmonic effects are ignored: In this case the maximum THD is (7.9%) which still exceeds the permissible limit (5%). The optimal capacitor placement results in considerable yearly benefits of $5557.5 which represents (20.1%) of the total cost of the system losses before capacitor placement.

Case 2) In this case the THD is controlled within prescribed limits (5%). The optimal capacitor placement results in considerable yearly benefits of $3116.8 (11.7% of the total cost of the system losses before capacitor placement), which are lower than that of Case 1.

In spite of that Case 1 achieves greater benefits than Case 2, but Case 2 is recommended. In Case 2 the limits of THD prevent possible harmonic amplification or resonance conditions which may result in considerable future cost caused by additional stress on equipment insulation, increased capacitor failure and interference with communication systems. Therefore, from a long-term point of view, the presented solution of Case 2 be more economical than that of Case 1.

CONCLUSIONS

In this paper, a binary particle swarm optimization is used for discrete optimization problem of optimal capacitor placement in the presence of nonlinear loads. The objective function is to maximize the net savings resulting from peak power and energy reduction, while taking capacitor cost into account. Maximum THD, maximum and minimum bus voltages and the number of allowed number of capacitor banks are considered as constraints. Using maximum THD as a constraint prevents the occurrence of harmonic parallel resonance. A comparison of results between the binary PSO and the exhaustive search indicates the effectiveness of binary PSO in finding improved solutions.

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