DISTRIBUTION SYSTEM AVAILABILITY ASSESSMENT – MONTE CARLO AND ANTITHETIC VARIATES METHOD

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ABSTRACT

The time sequential Monte Carlo simulation used in power systems for the estimation of reliability indices is a computationally expensive method. The accuracy of the results depends on the number of samples used in the simulation and the variance of the estimate. Variance reduction techniques can be employed to reduce the sample size needed to achieve a given precision in the estimate values. This paper discusses the Antithetic Variates application to the time sequential Monte Carlo simulation. The study cases are conducted on a small isolated distribution system with dispersed generation (DG).

INTRODUCTION

The major changes in distribution systems, due to the introduction of dispersed generation, make their operation schemes as well as their operational reliability to be modified. A re-evaluation of system reliability is therefore needed. The two often used approaches for power system reliability evaluation are the analytical [1] and simulation methods [1, 2]. Analytical techniques represent the system by a mathematical model, often simplified, and evaluate the reliability indices from this model using direct mathematical solutions. Simulation techniques estimate the reliability indices by simulating the actual process and random behaviour of the system and are generally more flexible when complex operating conditions and system considerations (bus load uncertainty, weather effects, etc.) have to be taken into account [3]. The type of simulation involving the sampling of values of stochastic variables from their probability distribution using random numbers is denoted as Monte Carlo simulation [1]. There are two basic techniques used in Monte Carlo simulation: sequential and non-sequential. The sequential simulation permits chronological issues to be considered and the reliability indices distribution calculation. The Monte Carlo simulation needs many trials to obtain a reasonable accuracy in the result of the estimat. Because of this, a special interest was assigned to reduce the number of samples needed for a given accuracy by means of variance reduction. A number of variance reduction techniques are used in power system reliability evaluation: Antithetic Variates, Control Variates, Importance Sampling, Stratified Sampling and Common Random Numbers (Correlated Sampling) [1, 4].
The sequential Monte Carlo simulation approach has been also used in other papers in combination with variance reduction techniques for reliability system evaluation [5]. The distinctive feature of our program is the simulation of the dynamic behaviour of the system, meaning that, along the time axis, we model not just one event but a sequence of events. During simulation, we can reach different system states, involving even the blackout and the restoration of the system. The reliability indices are computed for each system state and for each load bus. The application of a variance reduction technique becomes in this case more demanding and with not always encouraging results.
The paper describes modelling aspects and computational results of the Antithetic Variates method applied to time sequential Monte Carlo simulation for reliability evaluation of a small isolated distribution system with DG. The results were compared with the "natural" Monte Carlo simulation. Finally, alternative approaches to create negative correlation in the Antithetic Variates method are discussed.

MONTE CARLO SIMULATION

Time sequential Monte Carlo simulation

The time sequential Monte Carlo simulation used in this paper and usually employed to evaluate the system reliability indices involves the following steps [6]:
Step1. The simulation starts from a normal system state (all the components of the system are in the up state). A chronological hourly load curve gives the load variations for the simulated period (usually a year).
Step2. Generate random numbers for all the elements (generation, lines…) of the system and convert them into failure time (T_f) according to the failure probability distribution of each element. In this application, the times to failure are assumed to be exponentially distributed:

$$T_{f,k} = -\left(\frac{1}{\lambda_k}\right)\ln u_k \qquad k \in [1,...m] \qquad (1)$$

where m = number of equipments, λ = failure rate [frequency/yr], u = random number.
Step3. Compare the $T_{f,k}$ for all the elements. The minimum $T_{f,k}$ gives the next failure event and the failed element.
Step4. Generate a random number for the failed element and convert it in repair time (T_r) according to the repair probability distribution of the element. The times to repair are assumed to follow a Weibull distribution:

$$T_{r,k} = \alpha(-\ln u_k)^{1/\beta} \qquad k \in [1,...m] \qquad (2)$$

where α (scale parameter) depends on the mean time to repair and β (shape parameter) is equal to 6.

Step5. Simulate the first event. After each event, the system is analyzed, problems are identified (frequency, voltage and currents) and then, if possible, corrective actions are computed (including load shedding). The adequacy indices for each load bus are computed.

Step6. Return to **Step3** if the simulation time is less than a year. If the simulation time becomes greater than a year, calculate the indices for the whole year and go to **Step7**.

Step7. Calculate the expected value (E(F)) and the variance (V(F)) of the estimate function F for the wholes years:

$$E(F) = \frac{1}{N}\sum_{i=1}^{N} F(u_i) \tag{3}$$

$$V(F) = \frac{1}{N-1}\sum_{i=1}^{N} (F(u_i) - E(F))^2 \tag{4}$$

where N is the number of samples.

Step8. Repeat **Step1** to **Step7** until the coefficient of variation of the chosen reliability index becomes less than a tolerance level (TL). The coefficient of variation is [5]:

$$\varepsilon = \frac{\sqrt{V(F)}}{\sqrt{N}\cdot E(F)} \tag{5}$$

Figure 1 shows the described Monte Carlo procedure.

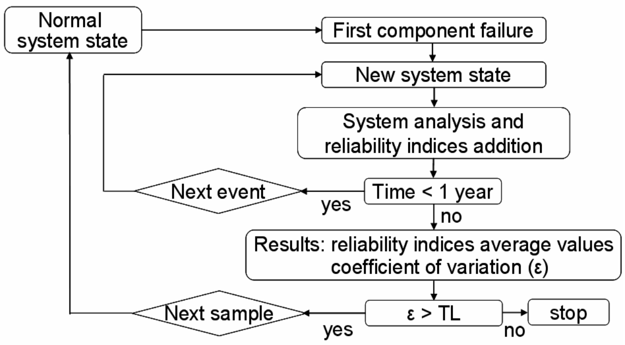

Fig.1: Monte Carlo simulation procedure

Variance reduction technique - Antithetic Variates method

Antithetic sampling has three highly desirable properties. First, it is easily to implement. Second, it may be used in conjunction with other variance reduction techniques because it changes only the random drawing procedure, not the actual estimators. Third, and most importantly, antithetic sampling requires no additional prior knowledge of the output random variables beyond monotonicity.

The main idea in this method is to try to create negative correlation between observations, generating one sample from the random numbers "u" and the antithetic one from the random numbers "1-u". If we consider F(u) and F(1-u), the response of samples "u" and "1-u", respectively, the new estimator and its variance are [4]:

$$E(F) = \frac{1}{2}[F(u) + F(1-u)] \tag{6}$$

$$V(E(F)) = \frac{1}{4}[V(F(u)) + V(F(1-u)) + 2\,\mathrm{cov}(F(u), F(1-u))] \tag{7}$$

By this technique, it is hoped to create negative correlation between the two responses. Negative correlation between responses is desirable since it decrease the variance of the new estimate response.

The main steps of the simulation are the same as in "natural" Monte Carlo. **Steps 1** to **6** are realized for "u" and then for "1-u". After each two antithetic samples the new estimator of the system E(F) (**Step7**) is calculated:

$$E(F) = \frac{1}{2N}\sum_{i=1}^{N}[F(u_i) + F(1-u_i)] \tag{8}$$

The coefficient of variation is computed according with equation (5) with: V(F)= variance of the new estimator E(F) and N= total number of samples divided by two.

Note that the computing time required by the equation (8) is twice the time required by the equation (3). Therefore, the estimator (8) is more efficient than the estimator (3) only if the variance corresponding to the former is smaller than half of the corresponding to the latter. This can be guaranteed if there is a monotonic relation between the system simulation response and the stochastic input variables [1]. In the simulation of complex systems, the response depends on a sequence of values of the stochastic input variable. Moreover, we can have K types of stochastic input variables, i. e. failure times, repair times, etc. Hence, in this case, it seems impossible to show analytically that there is a monotonic relation between the system simulation response and the input variables. So, it is difficult to recognize that the Antithetic Variates method could lead to a negative correlation between samples. If we take as reference [1], the best results are obtained when each input variable (failure time, repair time) has its own stream of random numbers. Also, another way to increase the desired negative correlation is to create a synchronisation between the two antithetic samples. If the j'th random number "u_j" (from one stream) generates a particular event, then, in the antithetic sample, ("1- u_j") should generate the same event [1].

PRACTICAL ASPECTS

Studied system

The studied system [7] shown in Figure 2 is a small low voltage distribution system with 204 customers, 52 fuel cells and 6 cogeneration units. This example is far from an actual distribution system but it presents several difficulties which are interesting for a reliability study. The total generating capacity is 1 480 kW. The system peak load is 1 053 kW. In order to represent the system loads variations, we use an hourly load curve which gives the load variations for 8 760 hours. The reliability data are outlined in Table I.

TABLE I: RELIABILITY DATA

Equipment	Fault rate (f/yr)	Mean time to repair (h/yr)	Stuck probability
Lines	10^{-1} (f/yr/km)	15	-
Breakers	10^{-5}	4	0.05
Generation units	10^{-4}	50	-

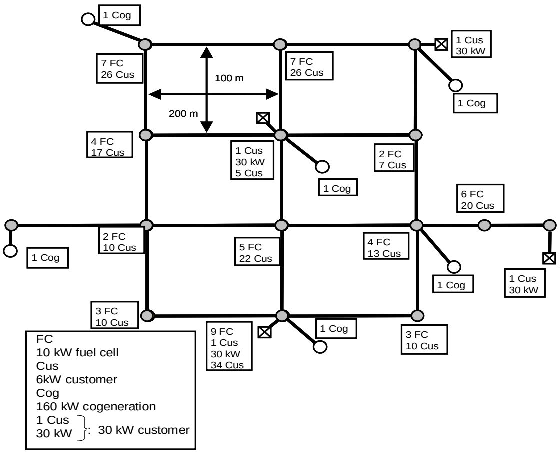

Fig. 2: The studied distribution system

System modeling [7]

To calculate the reliability indices for each new system state the topology of the system must be checked and some calculations must be done. First, the system is analyzed, problems are identified (frequency, voltage and currents) and then the system response is computed. At the end the interruption frequency and unavailability are calculated.

The system is analyzed through an AC load flow. After each disturbance, a new steady-state point of the system is calculated. If the system reaches a steady-state point (the load flow converges), voltages and currents are outputted. Then, problems of overloaded lines or voltage can be identified and corrective actions must be taken to bring back the system within acceptable limits. If the load flow diverges, this indicates that the system state would result in voltage stability problems. Voltage stability indicators can be computed to establish the critical points. In our application, the system is designed to shed load by customer's priority so, there is no need to know which nodes are critical.

System reactions are ensured by automatic equipments (spinning reserve, breakers, load shedding relays …) and remote controlled equipments (actuated by the system operator). In regard to the fault protection of the system, stuck probabilities are assigned to model the wrong functioning of the protection-breaker chain. The considered type of protection are the short-circuit protection, the overload protection and frequency relay. To model corrective actions performed by the operator, an optimization function is used. Based on the system state and equipments constraints, the function finds a suitable solution. In case of an overloaded line, active and reactive power settings can be modified at generator level to reduce the power flow and to prevent the line protections tripping. If it is not possible, priority load shedding is added. Finally the line protection will trip if the contingency cannot be avoided. To carry out these settings, the optimization is performed considering unit's active and reactive power limits. In the simulation, corrective actions are applied 2 minutes after the contingency. This time includes the contingency detection time, the time to perform a solution and the time to transmit the orders.

Test results

"Natural" Monte Carlo simulation

Initially, for estimating the reliability indices, the time sequential simulation approach without any variance reduction technique was used. The interruption frequency and the unavailability for each load bus are calculated. The simulation is stopped when the maximum coefficient of variation of the load buses becomes less than a tolerance level. The coefficient of variation imposed is 7% for the interruption frequency (interruption per year) and 12% for the unavailability (hour per year). After several simulations, we noticed an average value of about N=1000 sample years used to achieve the convergence. The simulation employed about 8200 seconds of CPU (central power unit) time. An example of result is outlined in Figure 3 (left curve).

Antithetic Variates method application

The above Antithetic Variates method was incorporated and tested in the time sequential simulation procedure to assess the adequacy of the system. Two random number streams, one for generating the times to failure and the other to obtain the times to repair, were used to create the synchronization between the two samples of antithetic simulation. So, in this case, the times to repair for all the elements of the system are generated at the beginning of the simulation and not after each fault.

An average value (between several simulations) of 400 sample years (N_{VA}=800 runs) were needed to achieve the same coefficients of variation regarding the reliability indices. So, an acceleration of about 20% is accomplished with this method. The Figure 3 shows the convergence of the interruption frequency for "natural" Monte Carlo simulation and using Antithetic Variates method (for one load bus of the system).

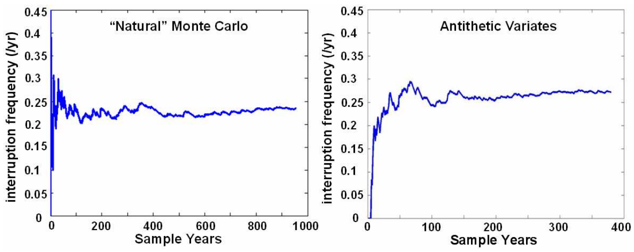

Fig. 3: Interruptions frequency Vs sample years

The Antithetic Variates used about 6500 seconds of CPU time to converge. That means also a reduction of about 20% in computational time. The obtained indices are individually analysed for each customer of the system. The error between the two simulations is rising till 7% for some dispersed customers (about 3% of loads). The system error (average value for all customers) remains usually under 2%.

The 20% acceleration obtained using the basic Antithetic Variates method was not a very satisfactory result in our opinion. Because of this poor performance in reducing the computational effort, other methods for creating negative correlation between the two observations were tested.

1. Like in the "natural" Monte Carlo, the first sample is created transforming the two vectors of random numbers U in times to failure and times to repair. In the antithetic sample, instead of using one minus the random numbers, it was used the absolute value of 2MTBF-T_f and 2MTBR-T_r. MTBF and MTBR are the mean time between failure and mean time between repairs, respectively. T_f and T_r are the failure and repair times used in the first sample. The simulation was repeated several times. The obtained results were not satisfactory, an average value of 1500 sample years were needed to achieve the same relative error regarding the reliability indices. In fact it was observed a little deceleration in results. It means that we have a positive correlation between the two antithetic samples instead of having a negative one. One possible explanation for these results can be that in the antithetic sample using directly 2MTBF-T_f and 2MTBR-T_r, the passage from the random variable into time is no more realized. The exponential probability distribution function is no more used in the antithetic sample to find the fault and repair times.

2. Another reduction of computational time was attempted by changing the simulation sample of one year to ten years. That means that the reliability indices for a sample are evaluated over ten years. Having a system that is very reliable, we expected to include more events in one sample. Also, we hoped to decrease the number of samples without events. Both "natural" Monte Carlo and the Antithetic Variates method were carried out for the decade sampling. The same coefficients of variation were imposed. First, we compared the simulation time using the year sampling and the decade sampling only for the "natural" Monte Carlo. The results were similar, about 8200 second of CPU time. Second, we compared the results using the decade sampling, for the "natural" Monte Carlo and applying the AV method and these results are presented in Figure 4.

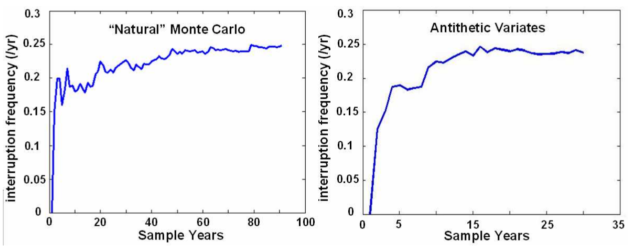

Fig. 4: Interruptions frequency Vs sample years

The "natural" Monte Carlo simulation used an average value of 90 sample decades to converge. The application of basic Antithetic Variates method ("u" and "1-u") leads to an average value of 30 (60 runs) sample decades meaning about 30% acceleration (obtained also in CPU time). Again, the improvements were not significant, indicating that neither this method has very much succeeded. However we can see a better negative correlation in the case of a decade sampling. Further experiments with other systems are required before this conclusion can be generalized.

CONCLUSION

Despite the fact it uses a significant computation time, the time sequential Monte Carlo simulation can readily produce solutions to complex problems and evaluate additional information such as probability distribution that are not realizable from analytical methods. Several variance reduction techniques are at our disposal to speed up the simulation time. One of these techniques, the Antithetic Variates, was experimented in this application.

The Antithetic Variates was chosen as a variance reduction technique because it is a straightforward method and it has the advantage of not disturbing the dynamic behaviour of the system, changing only the random drawing procedure. Unfortunately the system modelling complexity brought unexpected results. The computational experiments indicated that the basic Antithetic Variates scheme was not very attractive in our case, even if a good synchronization was realized between samples and each input variable had its own stream of random numbers. It seems that the negative correlation between samples was insufficient. A little progress was achieved changing the sample runs from one year to ten years without altering the calculated indices. Further experiments with other variance reduction schemes are required to obtain more powerful results.

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