

## DESIGNING SUBSTATION EARTHING GRID SYSTEM USING INTERVAL MATHEMATICS

Mohamed Anwar

Department of Electrical Engineering– Shoubra Faculty of Engineering, University of Benha- Egypt  
moh\_an1@yahoo.com

### ABSTRACT

*The paper presents the application of Interval Mathematics as a new method to, rigorously, address uncertainties associated with designing substation earthing grid system. While several methods exist to determine the important factors for personnel safety in and around substations such as earth resistance, touch, step and mesh potentials of substation earthing grid, these methods usually require data which may be uncertain in nature. To account for such uncertainties the interval mathematics is developed with the integration of input parameters' uncertainties, in interval format, into the governing standard expressions mentioned on IEEE STD 80-2000. The effects of uncertain inputs within the proposed model are examined for various assumed levels of overall uncertainties. To assess the relative contribution of each uncertain input, an interval sensitivity analysis is carried out. While catering for uncertainties, the method also offers utilities with alternatives for selecting the standard conductor size to be used. Successful implementation of the proposed method is described for the design and configuration arrangement of a 115/13 kV substation earthing rectangular grid with ground rods system.*

### INTRODUCTION

Earlier, design and implementation of substation earthing systems had been based on trial and error procedures, however, recently, computer- based design and analysis of substation earthing grid systems have evolved [1]-[4]. Satisfactory performance of substation earthing grid systems is critical under steady state and transient conditions [1]-[7]. E.g.; the impact of a poor or an improperly designed earthing system may range from faulty operation of electrical / electronic systems to injuries and death of personnel [2].

However, the data employed in the substation earthing grid safety analysis is usually derived from many sources with varying degrees of accuracy. Accounting for such uncertainties is necessary to produce realistic results which utilities can employ to make informed decisions regarding designing substation earthing grid system.

Uncertainties can be looked upon as a condition in which the possibility of errors exists as a result of having less than total information about the surrounding environment. They are beyond the utility's foreknowledge or control. Soil stratification can not be rule out of grounding system design [2]. There are cases where the top soil has a higher resistivity than the lower soil or the other way round [2]. So, the soil resistivity, crushed rock wet resistivity are always varying and it is not a realistic proposition to determine the important factors for personnel safety in and around substations such as earth resistance, touch, step and mesh potentials of substation earthing grid based on an average of their values as their values differs from time to

another, especially the soil resistivity, during the year according to the differing in temperature, pressure and humidity [1], [2]. Consequently, the validity of the results generated is questionable.

Interval mathematics provides a powerful tool for the implementation and extension of the "unknown but bounded" concept [8]-[10]. Using interval analysis, there is no need for many simulation runs as the total variation of the solution considers the simultaneous variations of all inputs in a single run. In this form of mathematics, interval numbers are used instead of single point numbers.

This paper presents the application of interval mathematics as a new method to address uncertainties associated with designing substation earthing grid system. Uncertainties in the parameters are integrated into the analysis, as interval numbers, determine the important factors for personnel safety in and around substations such as earth resistance, touch, step and mesh potentials of substation earthing grid. A comprehensive uncertainty level analysis is presented. The relative significance of each uncertain input is established through an interval sensitivity analysis. The method offers utilities with alternatives for selecting the standard conductor size to be used. In this study it is assumed that the system of ground electrodes has the form of a grid of horizontally buried conductors, supplemented by a number of vertical ground rods connected to the grid. Based on IEEE STD 80-2000, this concept represents the prevailing practice of most utilities both in the USA and in other countries. The proposed method is tested for the design and configuration arrangement of a 115/13 kV substation earthing rectangular grid with ground rods system and encouraging results are reported.

### THE GOVERNING EQUATIONS

In order to account for uncertainties associated with the substation earthing grid system design, the following analysis is followed [7]. The input parameters' uncertainties, in interval format, are integrated into the governing equations as follows:

$$R_g = r \left[ \frac{1}{L_T} + \frac{1}{\sqrt{20A}} \left( 1 + \frac{1}{1 + h\sqrt{20/A}} \right) \right] \quad (1)$$

where  $R_g$  is the interval grid resistance,  $\rho$  is the interval soil resistivity,  $L_T$  is the total effective length of buried conductor,  $A$  is the grid area, and  $h$  is the depth of the grid burial.

Ignoring the station resistance, the interval symmetrical ground fault current (assuming line to line to ground fault) is

$$I_f = \frac{3E}{3R_f + (R_1 + R_2 + R_o) + j((X_1 + X_2 + X_o))} \quad (2)$$

where  $E$  is the phase-to-neutral voltage,  $R_f$  is the estimated resistance of the fault (normally it is assumed  $R_f = 0$ ),  $R_1$ ,  $R_2$  are the positive and negative sequence equivalent system resistances respectively,  $R_o$  is the zero sequence equivalent system resistance,  $X_1$ ,  $X_2$  are the interval positive and negative sequence equivalent system reactances (subtransient)

respectively,  $X_0$  is the interval zero sequence equivalent system reactance.

The values  $R_1, R_2, R_0, X_1, X_2$ , and  $X_0$  are computed looking into the system from the point of fault.

So, the interval maximum grid current is given by

$$I_G = I_f * S_f * D_f \quad (3)$$

where  $D_f$  is the decrement factor and  $S_f$  is the division factor.

Assuming the use of copper wire and ambient temperature of 40 °C, the required interval conductor diameter (for fault duration  $t_f = 0.5$ ) in mm is [7]

$$d = \sqrt{\frac{4 * 3.569 I_f \sqrt{t_f}}{1000 \rho}} \quad (4)$$

Consequently, at this stage, the designer may opt to check if, alternately, the use of a less conductive (30%) copper-clad steel wire and the imposition of a more conservative maximum temperature limit of 700°C will still permit the use of a conductor with the above diameter  $d$ . So, the minimum interval conductor diameter to be used can be calculated by;

$$d_{min} = \sqrt{\frac{4 * 197.4}{1000 \rho \sqrt{\left(\frac{TCAP \cdot 10^{-4}}{t_c \alpha_r \rho_r}\right) \ln\left(\frac{k_o + T_m}{k_o + T_a}\right)}}} \quad (5)$$

where  $TCAP$  is the thermal capacity per unit volume,  $t_c$  is the current duration,  $\alpha_r$  is the thermal coefficient of resistivity at reference temperature,  $\rho_r$  is the resistivity of the ground conductor,  $T_m$  is the maximum allowable temperature, and  $T_a$  is the ambient temperature.

Assuming that for the particular station the location of grounded facilities within the fenced property is such that the person's weight can be expected to be at least 70 kg [7], the interval tolerable step and touch voltages for humans of 70 kg, respectively, can be computed as follows:

$$E_{step} = \frac{0.157(1000 + 6 C_s \rho_s)}{\sqrt{t_f}} \quad (6)$$

$$E_{touch} = \frac{0.157(1000 + 1.5 C_s \rho_s)}{\sqrt{t_f}} \quad (7)$$

where  $\rho_s$  is the interval crushed rock wet resistivity,  $C_s$  is the reduction factor and can be approximated as [7]

$$C_s = 1 - \frac{0.09 \left( \frac{r_s - r}{r_s} \right)}{2 h_s + 0.09} \quad (8)$$

where  $h_s$  is the thickness of crushed rock surfacing.

It is necessary to compare the interval ground potential rise ( $GPR$ ) to the interval tolerable touch voltage ( $E_{touch}$ ).  $GPR$  is calculated by

$$GPR = I_G * R_g \quad (9)$$

The interval mesh voltage ( $E_m$ ) at the center of the corner mesh is computed as follows [7]

$$E_m = \frac{r I_G K_m K_i}{L_C + \left[ 1.55 + 1.22 \left( \frac{L_r}{\sqrt{L_x^2 + L_y^2}} \right) \right] L_R} \quad (10)$$

where  $L_C$  is the total length of the conductor in the horizontal grid,  $L_R$  is the total length of ground rods,  $L_x$  and  $L_y$  are the length and the width of the substation respectively,  $K_i$  is the correction factor for grid geometry,  $K_m$  is the spacing factor for mesh voltage, and they are given by

$$K_m = \frac{1}{2p} \left[ \ln \left( \frac{D^2}{16hd} + \frac{(D+2h)^2}{8Dd} - \frac{h}{4d} \right) + \frac{k_{ii}}{k_h} \ln \frac{8}{(2n-1)} \right] \quad (11)$$

where :

$$K_h = \sqrt{1 + \frac{h}{h_o}}$$

$$n = n_a * n_b * n_c * n_d$$

$$n_a = \frac{2L_C}{L_p} \quad (12)$$

$$n_b = \sqrt{\frac{L_p}{4\sqrt{A}}}$$

$n_c = n_d = 1$  for rectangular grid

$$K_i = 0.644 + 0.148 n$$

where  $D$  is the equally grid spacing,  $K_{ii}$  is the Corrective weighting factor that adjusts for the effects of inner conductors on the corner mesh and equals 1 for grid with ground rods,  $K_i$  is the corrective weighting factor that emphasizes the effects of grid depth,  $h_o$  is the grid reference depth,  $n$  is the geometric factor composed of factors  $n_a, n_b, n_c$ , and  $n_d$ , and  $L_p$  is the peripheral length of the grid.

Finally, the interval Step voltage ( $E_s$ ) between a point above the outer corner of the grid and a point 1 m diagonally outside the grid is calculated as

$$E_s = \frac{r I_G K_s K_i}{0.75 L_C + 0.85 L_R} \quad (13)$$

$$K_s = \frac{1}{p} \left[ \frac{1}{2h} + \frac{1}{D+h} + \frac{1}{D} (1 - 0.5^{n-2}) \right]$$

where  $K_s$  is the spacing factor for step voltage.

## INTERVAL MATHEMATICS

Interval mathematics provides a useful tool in determining the effects of uncertainty in parameters used in a computation. In this form of mathematics, interval numbers are used instead of ordinary single point numbers. An interval number is defined as an ordered pair of real numbers representing the lower and upper bounds of the parameter range [9], [10]. An interval number can then be formally defined as follows;  $[a, b]$ , where  $a \leq b$ . In the special case where the upper and lower bounds of an interval number are equal, the interval is referred to as a point or a degenerate interval and interval mathematics is reduced to ordinary single point arithmetic.

Given two interval numbers,  $[a, b]$  and  $[c, d]$ , the rules for interval addition, subtraction, multiplication, and division are as follows:

$$[a,b] + [c,d] = [a+c, b+d]$$

$$[a,b] - [c,d] = [a-d, b-c]$$

$$[a,b] * [c,d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)] \quad (14)$$

$$[a,b] / [c,d] = [a,b] * [1/d, 1/c], \text{ where } 0 \notin [c, d]$$

Implementing interval analysis techniques confronts some obstacles because its algebraic structure is unlike that of common single point arithmetic. Accordingly, interval computations may produce wide bounds.

Given a set of interval input parameters, the bounds of the resulting interval computations may depend on the calculation

procedure as well as the input parameters. Therefore, an effort has to be made to reduce the width of the resulting interval bounds. Normally, the approach to producing better bounds has been to rearrange the expression to reduce the appearance of the interval parameters [9], [10].

## SIMULATION RESULTS

To illustrate the governing equations presented above, a 115/13kV substation grounding grid system, whose input data given in [7], is investigated. Choosing grid spacing  $D = 7$  m, for a rectangular  $63 \text{ m} \times 84 \text{ m}$  grid, the grid wire pattern is  $10 \times 13$ , and the grid conductor combined length is  $13 \times 63 \text{ m} + 10 \times 84 \text{ m} = 1659 \text{ m}$ . The depth of the grid burial is given as 0.5 meter, and reference depth of grid is 1 meter. Assume the use of 38 ground rods, each 10 m long, as shown in Fig. (1).

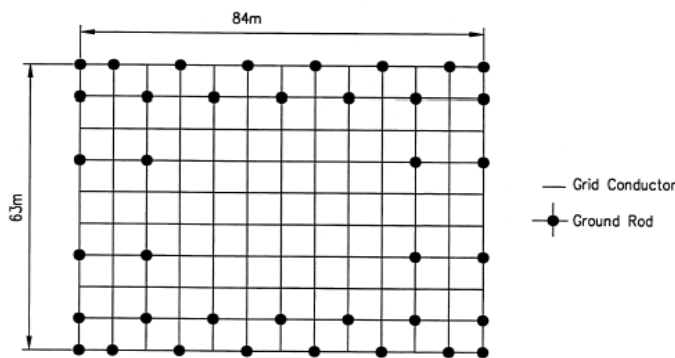


Fig. 1 Rectangular grid with 38 ground rods

The crushed-rock resistivity is assumed to be a conservative bound estimate based on actual measurements of typical rock samples. The equivalent system fault impedances and current division factor  $S_f$  are determined for the worst-fault type and location, including any conceivable system additions over the next 25 years. Thus, no additional safety factor for system growth is added [7]. In addition, it is assumed that the substation will not be cleared by circuit breakers with an automatic reclosing scheme. Thus, the fault duration and shock duration are equal [7].

The estimated values for  $\rho$ ,  $\rho_s$ ,  $X_1$ ,  $X_2$  and  $X_0$  are  $400\Omega\cdot\text{m}$ ,  $2500\Omega\cdot\text{m}$ ,  $10\Omega$ ,  $10\Omega$  and  $40\Omega$  respectively [7]. The following sections describe the compensation procedure for the test feeder; with the input parameters  $\rho$ ,  $\rho_s$ ,  $X_1$ ,  $X_2$  and  $X_0$  all assumed to be interval numbers. The computations are carried out using the Matlab toolbox [11].

### i) Base Case (5% tolerance)

To demonstrate the application of the proposed algorithm, the above equations are employed to obtain the required outcomes. A tolerance of  $\pm 5\%$  is assumed in all parameters.

Table I shows the values of the interval outcomes compared with the single point estimates values. It is clear that the estimated values of the outcomes are within the lower and upper bounds of the corresponding interval results.

The proposed interval technique furnishes utilities with alternatives of using any available standard conductor size, lying within the interval conductor size outcome. Prior knowledge of such information could be of significance in utility planning.

Table I  
Comparison between interval outcomes and single point estimate values

Interval Outcome	Tolerance 5%	Single point estimate
$I_G$ (A)	[1820.6, 2003.9]	1908
GPR (v)	[4529.1, 5509.5]	4995.8
$d$ (mm)	[4.49, 4.89]	4.67
$d_{\min}$ (mm)	[6.26, 6.81]	6.51
$E_{\text{step}}$ (v)	[2557.6, 2837.9]	2696.1
$E_{\text{touch}}$ (v)	[805.93, 875.98]	840.55
$E_{\text{mesh}}$ (v)	[690.92, 840.47]	762.11
$E_s$ (v)	[416.48, 506.63]	459.4
$R_g$ ( $\Omega$ )	[2.49, 2.75]	2.6185

### ii) Assessment of the uncertainty level

In order to assess the uncertainties associated with the various input parameters  $\rho$ ,  $\rho_s$ ,  $X_1$ ,  $X_2$  and  $X_0$ , the level of uncertainty of all parameters has been taken to vary by 10% in case and 15% in another. Table II shows the results of the interval outcomes for different uncertainty level. It is observed that the interval bounds of the different interval outcomes for the higher tolerances contains those of lower tolerances, e.g., the interval outcome of  $I_G$  for a 5% uncertainty is contained within the intervals of the 10% and 15% levels. It is also noted that the radii of the interval outcomes increase proportional to the increase of the uncertainty level. Results showed that all the interval outcomes (represented here by the interval midpoint) overestimate the single point estimate for all the different tolerances. As the interval outcome width increases (e.g. for 10% and 15% tolerances), the number of standard conductor sizes available, for use by utilities, increases.

Table II  
Results for different uncertainty levels

Interval Outcome	Tolerance 10%	Tolerance 15%
$I_G$ (A)	[1742.1, 2039.5]	[1668.1, 2228]
GPR (v)	[4105.6, 5874.4]	[3712.9, 6709.1]
$d$ (mm)	[4.34, 5.09]	[4.18, 5.42]
$d_{\min}$ (mm)	[6.03, 7.09]	[5.82, 7.54]
$E_{\text{step}}$ (v)	[2421.9, 2983.4]	[2288.8, 3133.5]
$E_{\text{touch}}$ (v)	[772.02, 912.36]	[738.73, 949.88]
$E_{\text{mesh}}$ (v)	[626.33, 896.14]	[566.4, 1023.5]
$E_s$ (v)	[377.55, 540.19]	[341.44, 616.95]
$R_g$ ( $\Omega$ )	[2.36, 2.88]	[2.23, 3.01]

### iii) Sensitivity analysis of the input parameters

By using interval analysis, there is no need for many simulation runs as the total variation of the solution considers the simultaneous variations of all inputs in a single run. In order to evaluate the relative influence of each input parameter  $\rho$ ,  $\rho_s$ ,  $X_1$ ,  $X_2$  and  $X_0$ , an interval sensitivity analysis has been carried out. It is clear that some input parameters have no influence on some interval outcomes, e.g.,  $X_0$  has no effect upon the interval  $d_0$  and  $\rho_s$  has no influence upon the interval GPR. Table III

shows the different interval outcomes when every input parameter is assumed to vary alone with tolerances of 5%, and 10%. Close inspection of Table III, reveals that  $\rho$  is the most influencing parameter on the interval GPR and interval  $E_s$  followed by  $X_0$  and finally  $X_1$  or  $X_2$ . The radius of the interval GPR, when varying  $\rho$  alone, is 249.8 and 499.6 for tolerance of 5 and 10% respectively, while the radius of the interval  $E_s$  is 22.97 and 45.94 for the same tolerances. As for varying  $X_0$  alone, these values are 159.7 and 320.4 respectively for interval GPR, 14.68, and 29.46 respectively for interval  $E_s$ . The other two parameters  $X_1$  and  $X_2$  have almost a small effect on interval GPR and interval  $E_s$  for the same tolerances. The above results point out to the importance of accurate determination of these parameters as the confidence in the computed interval outcomes depends mainly on the confidence in the input parameters and not on computational procedures.

**Table III**  
**Results for Sensitivity analysis of the input parameters**  
 (a)

Par.	$X_1$ or $X_2$		$X_0$	
	Tol. 5%	Tol. 10%	Tol. 5%	Tol. 10%
$I_G$ (A)	[1892.7, 1923.3]	[1877.8, 1938.9]	[1848.8, 1970.8]	[1793.3, 2038]
GPR (v)	[4956.2, 5036.1]	[4917.3, 5076.9]	[4841.2, 5160.6]	[4695.9, 5336.6]
$d$ (mm)	[4.66, 4.69]	[4.64, 4.72]	-----	-----
$d_{min}$ (mm)	[6.49, 6.54]	[6.46, 6.57]	-----	-----
$E_{mesh}$ (v)	[756.08, 768.25]	[750.14, 774.48]	[738.53, 787.25]	[716.37, 814.09]
$E_s$ (v)	[455.76, 463.09]	[452.18, 466.85]	[445.19, 474.55]	[431.83, 490.74]

(b)

Par.	$\rho$		$\rho_s$	
	Tol. 5%	Tol. 10%	Tol. 5%	Tol. 10%
GPR (v)	[4746, 5245.6]	[4496.2, 5495.4]	-----	-----
$E_{step}$ (v)	[2687.9, 2704.3]	[2679.7, 2712.5]	[2565, 2828.9]	[2435.3, 2963.5]
$E_{touch}$ (v)	[838.51, 842.59]	[836.47, 844.63]	[807.78, 873.73]	[775.36, 907.38]
$E_{mesh}$ (v)	[724.01, 800.22]	[685.9, 838.33]	-----	-----
$E_s$ (v)	[436.43, 482.37]	[413.461, 505.341]	-----	-----
$R_g$ ( $\Omega$ )	[2.49, 2.75]	[2.36, 2.88]	-----	-----

**CONCLUSIONS**

The designing of substation earthing grid system problem is modeled using interval mathematics method. Use of interval mathematics enables the integration of the effects of parameters uncertainties into the analysis and eliminates the need for many simulation runs. The effects of uncertain inputs within the proposed model are examined for various overall uncertainty levels. The relative contribution of each uncertain input is

assessed through an interval sensitivity analysis. While catering for uncertainties, the method offers utilities with alternatives for selecting the standard conductor size to be used. This enhances their ability to make informed decisions regarding designing substation earthing grid system. Successful implementation of the method is described using a 115/13 kV substation earthing rectangular grid with ground rods system.

**REFERENCES**

[1] M.Kang, C. Boo, H. Kim., J.Zurada, 2010, "Estimating Soil Parameters Using the Kernel Function", ICCSA 2010, Part II, LNCS 6017, 110–118.  
 [2] E. Ubeku, F.Odiase, 2009," Substation earthing grid safety analysis", *International journal of Electrical and Power Engineering*, 3 (5), 262-267.  
 [3] M.Loboda, R. Marciniak, 2009, " Reliability of earthing system as significant factor determining EMC, the power quality, and lightning protection", *I<sup>ère</sup> Conférence Nationale sur la Compatibilité électromagnétique, CNCEM'09*, Nov., 22-24.  
 [4] S. Ghoneim, H. Hirsch, A. Elmorshedy, R. Amer, 2007 "Optimum Grounding Grid Design by Using an Evolutionary Algorithm," *IEEE General Meeting, Tampa, Florida, USA*.  
 [5] L.Linden, 2006, "Design for interpreting the innovation occurring within a free/open source software development community", *Proceedings of the 2006 Southern Association for Information Systems Conference*,224-228.  
 [6] D. Garre'it, and H. Holley, "Calculation of Substation Ground Resistance Using Matrix Techniques" IEEE Transactions on Power Apparatus and Systems, vol PAS-99, no 5, Sept/Oct.1980, pp 2008-2011.  
 [7]IEEE STD. 80-2000, "Guide for Safety in AC Substation Grounding".  
 [8] L. V. Barboza, G. P. Dmuro, and R. H. S. Reiser, 2004, "Towards Interval Analysis of the Load Uncertainty in Power Electric Systems," *8th International Conference on Probabilistic Methods Applied to Power Systems, Iowa State University, Ames, Iowa, Sept. 12-16*.  
 [9]H. Shaalan, 2000 "Modeling uncertainty in electric utility economics using interval mathematics," in *Proc. of the 4th IASTED International Conference on Power & Energy Systems* Marbella, Spain., Sept., 675-678.  
 [10] R.E. Moore, 1966, *Interval analysis*. Englewood Cliffs, NJ: Prentice-Hall.  
 [11]S. M. Rump, 1990, *Intlab-Interval Laboratory, Developments in Reliable Computing*, (T. Csendes, ed.), Kluwer, Boston, pp. 77-104.