HARMONIC STATE ESTIMATION THROUGH OPTIMAL POWER QUALITY MONITORING

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ABSTRACT

The aim of this paper is to present a methodology that defines the most suitable places for installing power quality meters in power system networks, in order to guarantee that all state variables may be directly or indirectly monitored.

Initially, the methodology established connectivity rules based on Kirchhoff's Current and Voltage Laws that allows one to verify if all the state variables will be monitored by a possible monitoring system configuration.

Then, a branch-and-bound searching algorithm was developed, in order to determine the optimal monitoring system configuration for the IEEE 14-bus system.

The effectiveness towards the harmonic state estimation of the achieved monitoring systems configurations are checked with a time domain reference case.

INTRODUCTION

Harmonic state estimation methodologies are commonly based on single values decomposition techniques [1]. Basically, through such approaches not all nodal variables are considered for solving the linear system that corresponds to the power network being studied. Only some variables (the most relevant or the measured ones) are used. Thus, such approach introduces errors in the estimation processes and, depending on the system configuration, cannot be used.

It is possible to overcome such limitations through optimal monitoring. Optimal monitoring constitutes on defining the minimum number of monitors needed by a monitoring system, in such a way that it is able to monitor ("observe") all line currents and all bus voltages ("state variables") from a power system network. In order to do so, the buses and sections where these monitors should be installed have to be strategically selected. By selecting these buses and sections, one can calculate all the non-measured variables from the measured ones. Through this approach, the error sources for estimating the values for the state variables are only the line parameters accuracy and the monitoring system synchronization.

Normally, line parameters calculation does not represent a significant error source, due to the many developments accomplished over this topic in the past.

Synchronization of the meters acquisition in monitoring systems indeed represented a significant limitation towards the harmonic state estimation in the past. Nevertheless, due to recent developments in this field, the estimated error in the acquisition synchronization reached 5μs [2], allowing the reliable estimation of harmonic state variables.

Many different works about the optimal placement of power quality meters have already been published before. Among them, it is important to highlight [3]-[5], as they were the first ones to discuss such topic. In [5], the authors developed a statistical methodology that determines the buses that are more susceptible to capture the disturbances. Such buses are chosen as the most suitable places to install the power quality meters. Due to the statistical approach, one cannot guarantee that the meters installed at the positions determined by this methodology will be able to capture all disturbances. In [4], the configuration for the power quality meters take into consideration only voltage sags and swells. The methodology is based on the simulation of short-circuits, in order to characterize the power network response, regarding the voltage sag/swell occurrence. In [3], the authors define a methodology that analyses the network topology, in order to define the most suitable places to install the power quality meters. Such methodology was the most promising one, once it guarantees that the allocated meters should be able to monitor any type of disturbance.

The work presented in this paper is strongly based on the work presented in [3]. Some aspects in the modeling presented in [3] were changed in order to make possible its application on power networks. Similar to the work presented in [5], the search for the optimal configuration of the monitoring system was made through a branch-and-bound algorithm. However, due to the changes made in the original modeling, it is shown that a more intelligent search algorithm is required, in order to apply the methodology in real power networks.

In this work, the methodology is applied to the IEEE 14-bus system. Different solutions were obtained and commented. The effectiveness of the solutions was checked with a time domain reference case simulated using MATLAB Simulink ®.

METHODOLOGY

Monitoring System Configuration

The representation of the monitoring system configuration explicitly indicates the buses and the sections where the power quality meters should be installed.

Differently from what was assumed in [3], power quality meters normally have a limited number of channels. For example, according to the modeling presented in, if a power quality meter is installed in bus ***j*** in Figure 1, the currents at three different sections should be also measured ($\boldsymbol{i_{ij}}$, $\boldsymbol{i_{jk}}$ and $\boldsymbol{i_{jl}}$).

In the present methodology, such assumption is not necessary. The system configuration representation is slightly different, as shown in the following section.

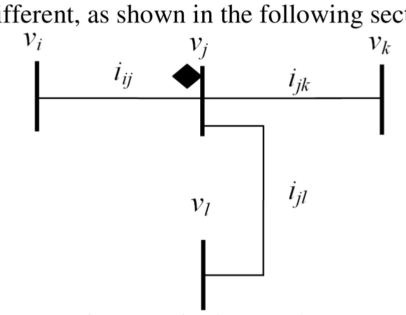

Figure 1 – Simple Network #1

Allocation Vector (AV)

The AV defines the configuration of the monitoring system. In this approach, the number of current channels is not pre-determined by the number of sections connected to the bus where the voltage should be measured. So, the size of the AV is defined by the summation of the number of sections connected at each bus. Equation (1) illustrates the size of the AV.

$$\text{Size}=\sum_{i=1}^{n}(\text{\# of Sections Connected at Bus } \boldsymbol{i}) \qquad (1)$$

Where:

- n is the number of buses in the power network.

Thus, each position in the vector represents a combination of bus and section in the power network where the meter could be installed.

The decision towards the installation or not of a meter measuring a specific pair of bus and section is represented by a binary variable, according to the definition shown in Equation (2).

$$av(i,j)=\begin{cases}1\text{, considering the installation of a meters at bus } \boldsymbol{i} \text{ and section } \boldsymbol{j}\\ 0\text{, considering the installation of no meters at bus } \boldsymbol{i} \text{ and section } \boldsymbol{j}\end{cases} \qquad (2)$$

In order to illustrate the AV, Figure 2 shows an example for the simple network illustrated in Figure 1. In this example, the meter is measuring the voltage at bus ***j*** and the current that flows through section ***jl***.

(v_i, i_{ij})	(v_j, i_{ij})	(v_j, i_{jk})	(v_j, i_{jl})	(v_k, i_{jk})	(v_l, i_{jl})
0	0	0	1	0	0

Figure 2 – Example of an *Allocation vector*

Objective-Function

It is possible to say that the cost of a monitoring system is composed by the cost of the hardware responsible for processing and storing the measurement data and the cost of the transducers that allow the data acquisition (PTs, CTs, etc.). Thus, in a simplified way, the cost of the monitoring system may be considered proportional to number of meters.

In order to minimize the number of meters required by a monitoring system, the objective function from this optimization problem becomes:

$$f_{OBJ}(AV)=\min\sum_{i=1}^{n}\sum_{j=1}^{m_n}av(i,j) \qquad (3)$$

Where:

- n is the number of buses in the power network;
- m_n is the number of sections connected and monitored at bus n.

Problem Constraints

Basically, the constraints indicate if a specific state variable could be calculated, considering the monitor installed at a specific bus. There are two types of constraints:

- based on Kirchhoff's Voltage Law;
- and based on Kirchhoff's Current Law.

Connectivity Constraint

The connectivity constraint is based on Kirchhoff's Voltage Law. Through this constraint it is possible to know (depending on the network connection and on the section's impedance) which are the voltages that can be directly calculated using the measured values. For example, assuming that the meter connected at bus ***j*** is measuring the voltage $\boldsymbol{v_j}$ and the current $\boldsymbol{i_{ij}}$ in Figure 1, the voltage at bus ***i*** can be directly calculated using the values the voltage $\boldsymbol{v_j}$, the current $\boldsymbol{i_{ij}}$ and the impedance from the section ***ij***.

In order to do so, it is necessary to build the Connectivity matrix (CM). Each row from the CM corresponds to a state variable (voltage or current), so the number of rows from the CM equals the number of state variables in the power network under analysis. Each column from the CM corresponds to a possible combination of bus and section that the power quality meter may be connected to, so the number of columns from the CM equals the size of the AV. Figure 3 illustrates the CM for the network shown in Figure 1.

Multiplying the CM by the AV, one obtains the Connectivity vector (CV). Each position from the CV indicates how many meters monitor the corresponding state variable. Equation (4) illustrates how the CV can be obtained.

$$CV = CM \times AV^{t} \qquad (4)$$

Redundancy Constraint

The redundancy constraint is also based on Kirchhoff's Voltage Law. It can be understood as an extension from the previous constraint. Figure 4 exemplifies its purpose.

Considering that the meters are measuring variables $\boldsymbol{v_i}$, $\boldsymbol{i_{ij}}$, $\boldsymbol{v_l}$ and $\boldsymbol{i_{kl}}$; variables $\boldsymbol{v_j}$ and $\boldsymbol{v_k}$ can be directly calculated, according to the previous constraint. As the values for variables $\boldsymbol{v_j}$ and $\boldsymbol{v_k}$ become known, one can calculate the value for variable $\boldsymbol{i_{jk}}$, through the voltage drop defined by $\boldsymbol{v_j}$ and $\boldsymbol{v_k}$, and the impedance of section ***jk***. Mathematically, such restriction can be written as illustrated in Equation (5).

$$\text{If } cv(\boldsymbol{v_j}) \geq 1 \text{ and } cv(\boldsymbol{v_k}) \geq 1, \text{ then } \boldsymbol{i_{jk}} \text{ can be monitored} \quad (5)$$

		(v_i,i_{ij})	(v_j,i_{ij})	(v_j,i_{jk})	(v_j,i_{jl})	(v_k,i_{jk})	(v_l,i_{jl})	
	...	...	...	...	...	...	...	...
v_i	...	1	1	0	0	0	0	...
v_j	...	1	1	1	1	1	1	...
v_k	...	0	0	1	1	0	0	...
v_l	...	0	0	0	0	1	1	...
i_{ij}	...	1	1	0	0	0	0	...
i_{jk}	...	0	0	1	1	0	0	...
i_{jl}	...	0	0	0	0	1	1	...
	...	...	...	...	...	...	...	...

Figure 3 – Connectivity Matrix

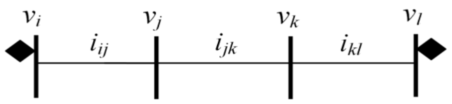

Figure 4 – Simple Network #2

The Redundancy constraint is represented through two different matrices, the Redundancy-From matrix (RFM) and the Redundancy-To matrix (RTM). The RFM and the RTM are similar to the CM, i.e., the number of rows from the RFM and the RTM equals the number of state variables in the power network and the number of columns from the RFM and the RTM equals the size of the AV. Equations (6) and (7) illustrate how the RFM and the RTM can be obtained.

$$rfm\left(i_{jk},:\right) = \begin{cases} cm\,(v_j,:) & \text{, if bus } \boldsymbol{j} \text{ is connected to bus } \boldsymbol{k} \\ 0 & \text{, if bus } \boldsymbol{j} \text{ is NOT connected to bus } \boldsymbol{k} \end{cases} \quad (6)$$

$$rtm\left(i_{jk},:\right) = \begin{cases} cm\,(v_k,:) & \text{, if bus } \boldsymbol{j} \text{ is connected to bus } \boldsymbol{k} \\ 0 & \text{, if bus } \boldsymbol{j} \text{ is NOT connected to bus } \boldsymbol{k} \end{cases} \quad (7)$$

One can notice that such constraint is applied to the current state variables only. Thus, the rows that correspond to the voltage state variables are all filled with null values.

Multiplying the RFM and the RTM by the AV, one obtains the Redundancy-From vector (RFV) and the Redundancy-To vector (RTV), respectively. Equations (8) and (9) illustrate how the RFV and RTV can be obtained.

$$RFV = RFM \times AV^{t} \quad (8)$$

$$RTV = RTM \times AV^{t} \quad (9)$$

One specific current state variable will be monitored according to the Redundancy constraint only when the positions of the corresponding state variable in both vectors are not null. Mathematically, such condition can be represented through the Redundancy Vector (RV), which is the intersection of RFV and RTV:

$$RV = RFV \cap RTV$$
$$\text{So, if } RFV(\boldsymbol{i_{jk}}) \geq 1 \text{ and } RTV(\boldsymbol{i_{jk}}) \geq 1, \quad (10)$$
$$RV(\boldsymbol{i_{jk}}) = 1, \text{ then } \boldsymbol{i_{jk}} \text{ can be monitored}$$

Co-connectivity Constraint

The Co-connectivity constraint is based on Kirchhoff's Current Law. It can be divided into two different sub-constraints, according to the bus type:

- Co-connectivity Constraint for Buses with Loads or Generations;
- Co-connectivity Constraint for Buses without Loads or Generations.

Through this constraint, the voltage at a specific bus can be determined if the voltages at the buses from all the sections connected to this specific bus are known. Considering Figure 5, Equation (11) illustrates the evaluation made through these constraints.

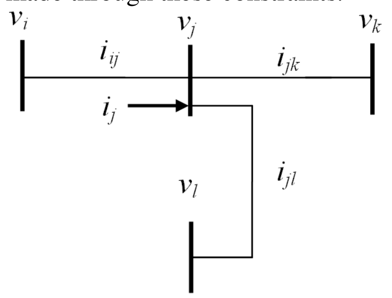

Figure 5 – Simple Network #3

$$i_{ij} + i_{jk} + i_{jl} + i_j = 0 \Rightarrow \frac{(v_i - v_j)}{z_{ij}} + \frac{(v_k - v_j)}{z_{jk}} + \frac{(v_l - v_j)}{z_{jl}} = -i_j \quad (11)$$

When there is a generator or load connected at bus $\boldsymbol{j}$, one should necessarily know its voltage x current characteristic, in order to define the injected current $\boldsymbol{i_j}$. When there is neither generator nor load connected at bus $\boldsymbol{j}$, current $\boldsymbol{i_j}$ equals zero.

Such constraint is represented through the Co-connectivity Vector (CCV). Basically, in order to verify if this constraint is satisfied; the CV should be analyzed first. Thus, the positions from the CCV are filled according to the position from the CV. Equation (12) illustrates the procedure for filling the CCV that corresponds to Figure 5.

$$ccv\,(v_j) = cv\,(v_i) \cdot cv\,(v_k) \cdot cv\,(v_l) \quad (12)$$

Optimization Process

The optimal meters allocation was made through a branch and bound algorithm. Basically, at each node from the process tree, the corresponding configuration for the monitoring system (the AV) was assessed, as shown in Figure 6. When all restrictions were satisfied, i.e. when the meter allocation allows the monitoring of all state variables, the optimization process was stopped.

In order to verify if all restrictions were satisfied, the union between the CV, the RV and the CCV should be executed. If the Final Vector (FV) presents no null position, the corresponding configuration monitors all state variables. Equation (13) illustrates the verification process.

$$FV = CV \cup RV \cup CCV$$
$$\text{So, if } FV(k) \geq 1 \forall\, k, \quad (13)$$
$$\text{then } \textbf{all state variables} \text{ can be monitored}$$

RESULTS

The optimal power quality meter allocation was applied for the IEEE-14 bus system [6]. No knowledge over the behavior of the generators and loads was assumed. The optimization process determined different configurations alternatives for the monitoring system. One of them is illustrated in Table I.

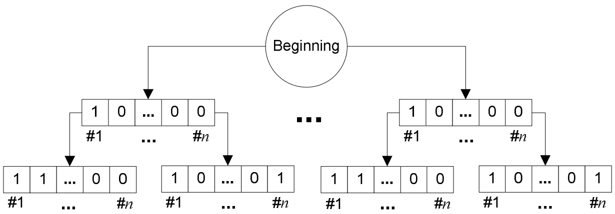

Figure 6 – Tree Structure for Branch and Bound searching Process

Table I – Monitoring System Configuration

Buses To Be Measured	Sections To Be Measured	
	From Bus	To Bus
Bus #2	Bus #1	Bus #5
Bus #5	Bus #2	Bus #3
Bus #6	Bus #4	Bus #9
Bus #8	Bus #6	Bus #12
Bus #9	Bus #7	Bus #8
Bus #10	Bus #10	Bus #11
Bus #14	Bus #13	Bus #14

In order to execute the harmonic state estimation, the IEEE-14 bus system was simulated using MATLAB Simulink®. Such modeling was used as reference for the estimation process. Due to space limitations, only two loads injecting distortions at the 3rd Harmonic Order were considered for the original circuit. The parameters for the harmonic loads were listed in Table II.

Table II – Harmonic Loads – 3rd Harmonic Order

Connection Bus	Magnitude [pu]	Phase Angle [º]
Bus #4	0.50	-45.0
Bus #14	0.25	75.0

So, the values for voltages at the buses and for the currents through the sections listed in the Table I were used as "measurements" for the estimation process. In other words, the voltages and currents for the buses and sections listed in Table I were obtained through the MATLAB Simulink® simulation. Through these values, the harmonic voltages and currents for the other buses and sections were calculated using Kirchhoff's Current and Voltage Laws. A comparison between the real values (the ones obtained through the MATLAB Simulink® simulation) and the estimated ones is presented in Tables III and IV.

CONCLUSIONS

A method for determining optimal location of power quality meters was studied and extended. Some modifications in the original methodology were introduced, in order to make possible the use of such methodology in real power systems.

The methodology proved to be highly effective for estimating harmonic state variables. The highest error obtained was around 0.5%, which is probably due to different number truncations during the calculation process.

Table III – "Measured" and Estimated Values - Bus Voltages

	Measured		Estimated		Error [%]
	Mag. [pu]	Phase [º]	Mag. [pu]	Phase [º]	
Bus #1	0.000	-103.3	0.000	2.3	0.0000%
Bus #3	0.104	-26.3	0.104	-26.3	0.0002%
Bus #4	0.174	-9.5	0.174	-9.5	0.0002%
Bus #7	0.311	4.9	0.311	4.9	0.0001%
Bus #11	0.289	19.1	0.289	19.1	0.0001%
Bus #12	0.289	27.2	0.289	27.2	0.0001%
Bus #13	0.295	27.6	0.295	27.6	0.0000%

Table IV – "Measured" and Estimated Values – Section Currents

		Measured		Estimated		Error [%]
From Bus	To Bus	Mag. [pu]	Phase [º]	Mag. [pu]	Phase [º]	
#1	#2	0.000	-56.6	0.000	-81.4	0.0000%
#2	#4	0.327	86.7	0.327	86.7	0.0675%
#2	#5	0.257	94.0	0.257	94.0	0.0383%
#3	#4	0.155	109.9	0.155	109.9	0.1460%
#4	#5	0.344	-116.6	0.344	-116.6	0.1327%
#4	#7	0.238	111.9	0.238	111.9	0.0476%
#5	#6	0.243	141.7	0.243	141.7	0.0470%
#6	#11	0.084	37.8	0.084	37.8	0.0871%
#6	#13	0.032	87.6	0.032	87.7	0.2357%
#7	#9	0.069	-177.5	0.069	-177.4	0.1997%
#9	#10	0.057	-119.3	0.057	-119.3	0.0284%
#9	#14	0.160	-163.9	0.160	-163.9	0.0435%
#12	#13	0.010	153.8	0.010	153.8	0.4438%

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