

## DISTRIBUTED OPTIMIZATION-BASED CONTROL OF ELECTRICAL DISTRIBUTION SYSTEMS WITH ACTIVE DISTRIBUTED RESOURCES

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### ABSTRACT

*This paper describes a framework to support the electricity distribution system operator (DSO) in identifying the appropriate control strategy in a system due to the stochastic nature of the integrated intermittent distributed generation. A distributed optimization and control algorithm is applied in order to determine the look-ahead allocation of distributed resources for risk mitigation, as well as real-time controls to attempt to ensure that the electrical distribution system (DS) is managed in a reliable and cost-effective manner. The management of these active resources will thus ensure that the DS can be optimized to operate in an efficient manner while providing adequate security.*

### INTRODUCTION

The progression in the development of modern electricity grid infrastructure is leading to rapid changes in the structure and operation of this critical system. System performance may be drastically altered due to the incorporation of new sources of renewable electrical generation within the DS, as well as the development of flexible, demand-responsive customers and distributed generation. In addition to these changes, developments in electricity markets and communications technologies enable increasing customer participation within the electricity markets through response to technical and economic signals.

In response to these system developments, it will be critical to develop and implement advanced strategies for control of the system to ensure that it is operated in an economically efficient and secure manner. Risks posed to the system, such as substation transformer overload, line thermal limitations, and defined voltage constraints are amongst the set of potential risks which must be accounted for in the development of an operational plan for the network resources. While planning the deployment of distributed generation (DG) or demand response (DR) resources, further complications are encountered when accounting for the inherent stochasticity of renewable generation sources. The

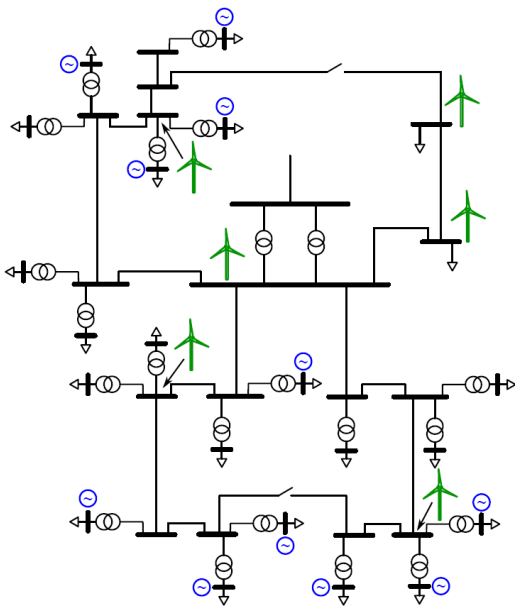
probabilistic uncertainty of these energy sources must be accounted for when optimizing system resource plans, which leads to significant increases in computational complexity. Further, once an adequate plan is developed, the real-time deployment of the resources poses an additional problem, since the uncertainty of renewable sources will dictate variations in the real-time utilization of the allocated resources.

To address this set of challenges, a framework is presented in this paper which seeks to guide both operational planning and real-time control of the system.

### PLANNING & CONTROL FRAMEWORK

The optimizations involved in developing an operational plan for a DS with the inclusion of demand-responsive customers and intermittent renewables present a significant computational challenge. The requirement of using a nonlinear multi-hour AC optimization to model both power flows and voltage levels in the system further complicates simulation, and motivates the use of advanced techniques to accelerate optimization.

The framework proposed in this paper implements a multi-stage operational planning and control strategy to address the previously mentioned computational and system modelling challenges. In the first stage, the DS and its resources are aggregated through the use of a network reduction algorithm which seeks to preserve a desired level of accuracy in network simulations while significantly reducing the amount of computation required. The second stage utilizes the aggregated network as an input, in addition to a characterization of the stochastic behaviour of the incorporated renewable generation, in order to perform a probabilistic AC optimal power flow (ACOPF) to guide the operational planning of the system resources. The third stage of the framework utilizes both the aggregated network from the first stage and the probabilistic results from the second stage as inputs to a MAS-based control scheme which determines real-time allocation of resources in the network.



**Figure 1:** Original 58-bus distribution network, including locations of renewable wind generation and potential distributed generation (which could also be in the form of demand-responsive curtailment.)

**Network & Resource Aggregation**

When performing an optimization to determine the operational plan for a DS, it is usually necessary to consider both the power flows which may lead to thermal stress, and to constrain voltage levels within a prescribed range to preserve voltage stability. This requires the use of a nonlinear ACOPF optimization, and as the number of nodes  $N$  in the simulated network increases, the number of nonlinear equations required to simulate the network increases as  $N^2$ . In many cases, this can lead to unreasonable simulation times.

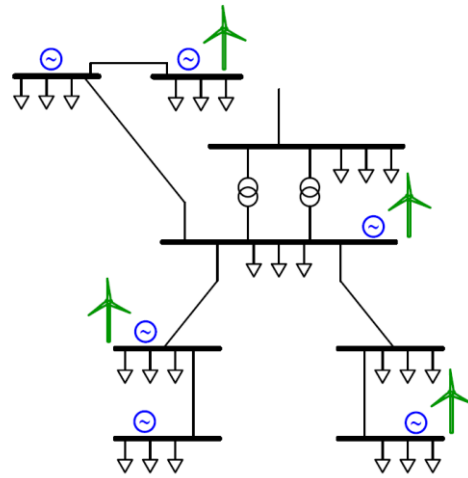
A method of addressing this is to perform an aggregation of the network to condense both the physical network and the nodal resources, as seen in Fig. 1, into a new  $N_p$  node network, where the nodal loads and generation are collected and placed proportionally at the appropriately locations. This aggregation is enabled through the construction of a permutation matrix  $P$ , and a condensation matrix  $C$ , which provide a mathematical description of the nodal aggregation, and whose formation is described in the literature [1,2]. The loads are then condensed as:

$$S_{load,R} = P_{load,R} + jQ_{load,R} = C \cdot P \cdot S_{load}, (1)$$

$$S_{gen,R} = P_{gen,R} + jQ_{gen,R} = C \cdot P \cdot S_{gen}, (2)$$

Where subscript 'R' indicates nodal quantities from reduction, and a new admittance matrix is defined as  $Y_A$ .

Once aggregated, two sets of compensating loads are developed which assist in preserving accuracy during optimization, as seen in Fig. 2. The first compensating load,  $P_{diff}$ , is designed to compensate the aggregated network in the base case, and the second load,  $P_{\Delta}$ ,



**Figure 2:** Aggregation of the 58-bus network yielding an 8-bus equivalent.

compensates when any renewable generation or demand response differs from the base case.

A typical ACOPF optimization can then be applied, where all the nodal contributions in the aggregated system are combined to form

$$P_{opt,i} = P_{load,R,i} + P_{gen,R,i} + P_{shift,i} + P_{curt,i}, (3)$$

$$Q_{opt,i} = Q_{load,R,i} + Q_{gen,R,i} + Q_{shift,i} + Q_{curt,i}, (4)$$

and the linear constraints ensuring that nodal power flows sum to zero are modified to include the impact of the two compensating loads which are present at each node in the aggregated network

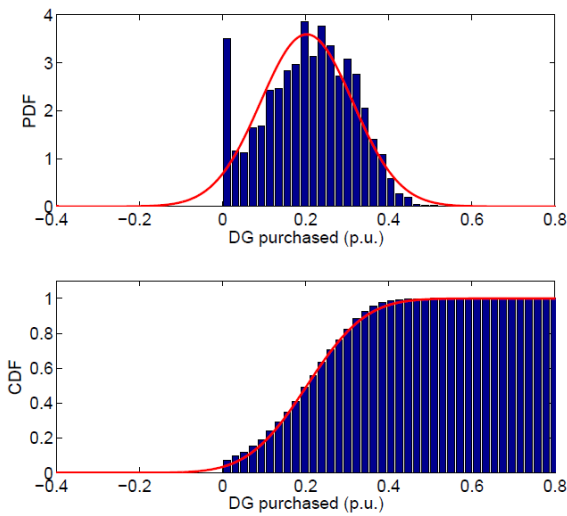
$$P_{opt,i} + P_{diff,i} + P_{\Delta,i} = 0, (5)$$

$$Q_{opt,i} + Q_{diff,i} + Q_{\Delta,i} = 0. (6)$$

**Probabilistic Optimal Power Flow**

Though system complexity has been significantly reduced via the use of an aggregation method, system simulation and optimization is still significantly complicated by the inclusion of the stochastic behaviour of the renewable generation within the DS, in this case the presence of wind generation. A traditional method for assessing the impact of the generation stochasticity is the utilization of a Monte Carlo (MC) simulation method to collect statistics for the system optimization outputs. In this case the outputs considered are the required allocation of DR and DG to address system risks under generation uncertainty. An alternative probabilistic method, Point Estimate Methods (PEM) [3,4], require a significantly reduced amount of computation to yield the statistical properties of the outputs, and are shown to compare well to MC results.

Application of a PEM proceeds by creating a set of input data referred to as concentrations. Each of the concentrations yields two components, the first being a point within the random variable input space  $p_{j,k}$ , and the



**Figure 3:** Distributions (PDF and CDF) of required DG within the aggregated network for a chosen hour.

second being a weight corresponding to the point,  $w_{j,k}$ .

While the MC method utilizes random sampling of the input variables to perform the simulation, a PEM selects a critical set of input points, and applies the appropriate weights to solve for the outputs. For each of the input points from the PEM concentrations, an optimization is performed to calculate an output variable

$$z_{j,k} = f_c(\mathbf{v}, \boldsymbol{\theta}, \mathbf{p}_{j,k}), \quad (7)$$

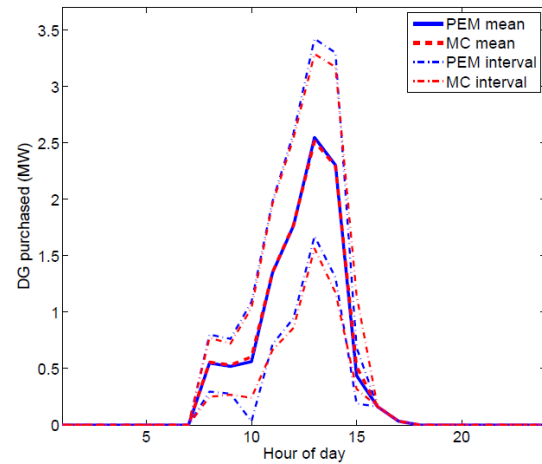
which is the result of an individual ACOPF with objective function  $f_c$ , and the data from all concentrations is used to calculate the statistical moments of the output as

$$E[z^n] = \sum_{j=1}^{N_V} \sum_{k=1}^K w_{j,k} (z_{j,k})^n. \quad (8)$$

Significant computational savings can be achieved through the application of a PEM method to determine the statistical properties of operational planning outputs, such as the amount of DG which may need to be used to ensure secure and cost-effective system operation. Figure 3 displays both the probability distribution function (PDF) and cumulative distribution function (CDF) of the DG requirements for a single hour, where the PEM distribution is compared to a histogram of the MC result. Additionally, the results of a 24-hour optimization to determine DG requirements are shown in Fig. 4, comparing the accuracy of the PEM results to those of a MC simulation.

### Multi-Agent System for Autonomous Control

The goal of the initial two stages of the proposed framework was to analyse the network to gain insight on the impact which the uncertain generation may have on the DS, and to ascertain the probabilistic distributions of the resource allocations which would be required to ensure secure and cost-effective operation. The results of the probabilistic OPF applied to the aggregated network



**Figure 4:** Statistics of the DG quantities required under the wind generation uncertainty for the simulated 24 hour period. Both a comparison of the mean and 90% confidence intervals are shown.

are utilized to develop a look-ahead operational plan, but an additional control strategy is required to execute the plan in real-time.

A multi-agent control system [5,6] is a desirable alternative for the coordination of the DS resources, as it requires infrequent communication to properly allocate system resources, since it requires only localized system information. This property may also assist in making the control more resilient to both physical and communications disturbances.

The MAS controls proceed by utilizing the aggregated network to form a directed graph, as shown in Fig. 5, based on the physical structure of the aggregated admittance matrix of the DS. Each node from the aggregated network is represented by a vertex in the graph, and for any nodes which share a connection in the admittance matrix, edge connections are established in both directions between the vertices, and a self-directed edge is placed on each vertex.

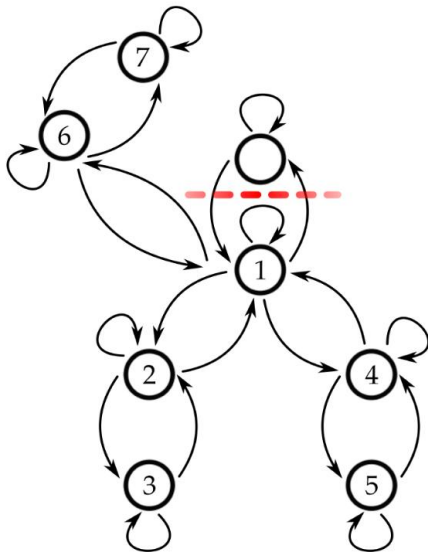
Two sets of edge weights are applied for control of the system and stored in matrices  $P_\mu$  and  $P_\sigma$ , which are used respectively to control the division of allocation amongst vertices and to ensure the sum of allocated resources equals the total requested resources. A set of numerator and denominator coefficients are iteratively defined as

$$\mu(k+1) = P_\mu \cdot \mu(k), \quad \mu(0) = x - \pi^{min}, \quad (9)$$

$$\sigma(k+1) = P_\sigma \cdot \sigma(k), \quad \sigma(0) = \pi^{max} - \pi^{min}, \quad (10)$$

where  $x$  is the initial vector of requested resources, and  $\pi^{max}$  and  $\pi^{min}$  are respectively the vectors of maximum and minimum resource allocation allowed at each node.

The control algorithm proceeds with each vertex calculating its own resource contribution, and passing a remaining portion, dependent upon the edge weight matrices, to its neighbouring nodes. The iterative computation of the individual vertex contribution is



**Figure 5:** Directed graph for the multi-agent system control, derived from the structure of the aggregated network of Fig. 2.

$$\pi_j(k) = \pi_j^{\min} + \frac{\alpha \mu_j(k)}{\sigma_j(k)} (\pi^{\max} - \pi^{\min}), \quad (11)$$

which, when performed successively throughout the vertices within the graph (either synchronously or asynchronously), succeeds in distributing the requested resources (in the case of the DS, the desired resource being some quantity of DG, DR, or reactive compensation.)

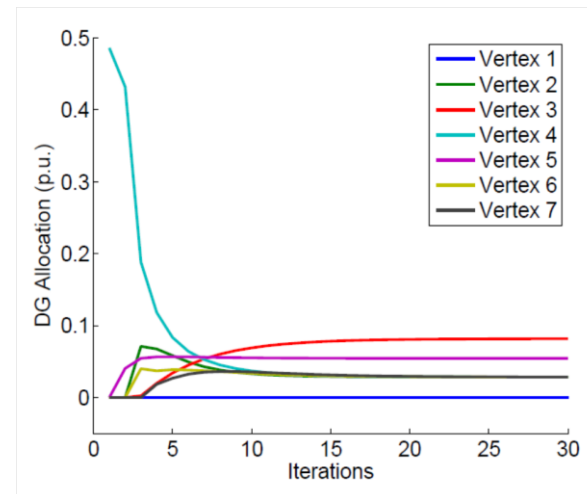
An example of the application of the MAS control scheme applied to the given aggregated network is shown in Fig. 6. In this case, two of the vertices are prescribed by the edge weight matrices to assume more of the requested DG load, vertex 1 is set to allocate no DG, and the remaining vertices should split the remainder of the requested resources evenly. It is seen that the algorithm appropriately addresses the autonomous distributed allocation of the requested resources.

## CONCLUSIONS

The framework proposed in the paper provides an effective methodology for the analysis of a DS while considering the impact of renewable generation uncertainty, allowing for the development of an operational plan which achieves security and economic efficiency. Subsequently, the multi-agent control architecture is shown to utilize the planning results to effectively implement distributed control of resources.

## ACKNOWLEDGEMENTS

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**Figure 6:** Iterative convergence of the solution for the MAS-based distributed algorithm. Note that the MAS control intentionally divides the resources amongst nodes based on desired weighting.

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