ANALYSIS OF A CONVOLUTION METHOD FOR THE ASSESSMENT OF DISTRIBUTION GRIDS WITHIN PROBABILISTIC POWER FLOW CALCULATION

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ABSTRACT

Probabilistic power flow calculation techniques are used in order to assess uncertainties due to current or future states of distribution grids (DG). These uncertainties are the stochastic behaviour of distributed energy resources (DER) and their hardly predictable number, installed capacity and location. This paper presents a consistent and comprising modelling of DER in form of parameterized probability density functions (PDF) and a convolution method, which takes into account correlations between DER and minimizes linearization errors by using a multilinearization approach.

INTRODUCTION

Convolution approaches in probabilistic power flow calculation have been discussed in literature [1-6]. The central idea is to linearize the power flow equations at one ("linearization") or several ("multilinearization", [1-2]) operating points, treat the input variables as probability density functions and determine the output probability density functions by a weighted convolution process. This paper proposes a novel way of probabilistic modelling of DER considering domestic households, electric vehicles and photovoltaic (PV) units and shows the achieved results for example cases, where the influence of correlations between input variables and errors due to linearization are discussed.

MODELLING OF DER

Households (HH)

To analyse the power demand of households, smart meter data of 39 households from the years 2010 to 2012 has been analysed, which represent the 15 minute mean power values of domestic households. The power values were normalized to the annual energy demand (AED in MWh) of each household ($p_{HH,n}$) and separated by means of season (winter, transition and summer), daytype (workday, Saturday, Sunday) and the 15-minutes timestep of the day (TOD) according to Table 1.

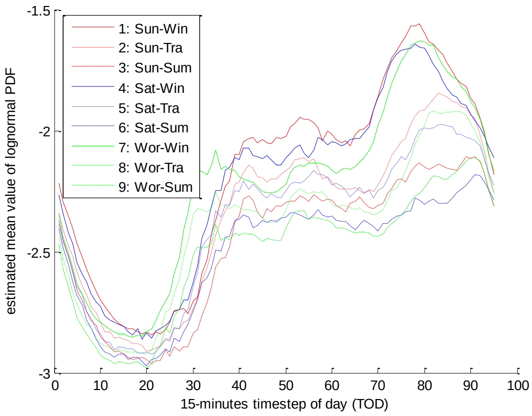
Daytypes (DT)		season		
		Win	Tra	Sum
Type of day	Sun	1	2	3
	Sat	4	5	6
	Wor	7	8	9

Table 1: Enumeration of daytypes

These normalised power values for each DT-TOD combination was estimated with a maximum likelihood estimate for a lognormal PDF, which showed to be the best estimate for the distribution of normalised smart meter data and has the form:

$$f(p_{HH,n}) = \frac{1}{p_{HH,n}\sigma_{HH}\sqrt{2\pi}} e^{\frac{-(\ln p_{HH,n}-\mu_{HH})^2}{2\sigma_{HH}^2}},$$

where μ_{HH} and σ_{HH} are mean and standard deviation of this lognormal PDF and functions of DT and TOD as shown in Figure 1.



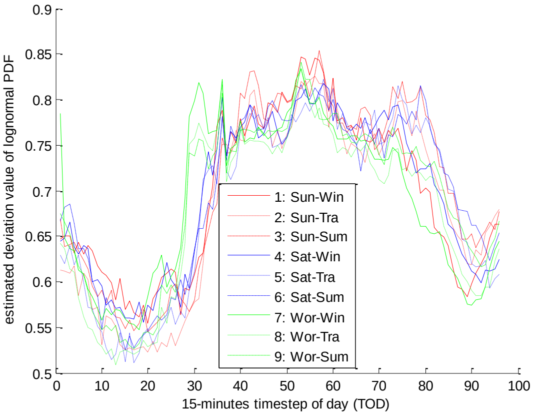


Figure 1: μ_{HH} (top) and σ_{HH} (bottom) as functions of DT and TOD

σ_{HH} is between 0.5 and 0.7 during the evening and night and between 0.7 and 0.85 during the day, almost independent of the daytype. μ_{HH} almost has the form of standard load profiles and is between -3 and -1.5, where the peak occurs in winter evenings.

By de-normalization with a certain AED one can get the lognormal PDFs as a function of TD, TOD and AED. Changing the TD from 1 to 2 shifts the lognormal PDF to the left, changing the TOD from 60 to 80 or the AED from 1 to 3 MWh shifts the lognormal PDF to the right (Figure 2).

In principle the modelling for reactive power can be done accordingly. In the following, a constant power factor of 0.95 is assumed for the reactive power of HH.

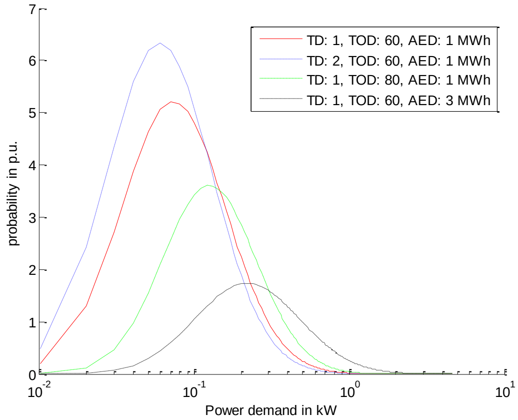


Figure 2: Variation of the lognormal PDF while changing the values of TD, TOD and AED

Photovoltaics (PV)

The behaviour of DER can be described by means of two distributions: On the one hand the peak power distribution (PPD) describes the frequency of occurrence of installed power capacities of DER. On the other hand the normalized power performance (NPP) describes the probability that a PV unit provides a certain normalized power. The NPP is a time dependent function and comprises all the interior characteristics of the DER. The combination of PPD and NPP describes all the information of a DER needed for a convolution process.

Figure 3 shows the PPD of small PV units in Germany according to [7]. Frequent installed peak powers are around 5 kW and 30 kW. The approximated curve was derived by modelling the PPD as a superposition of two lognormal functions with peaks at 5 and 30 kW similar to the modelling of households.

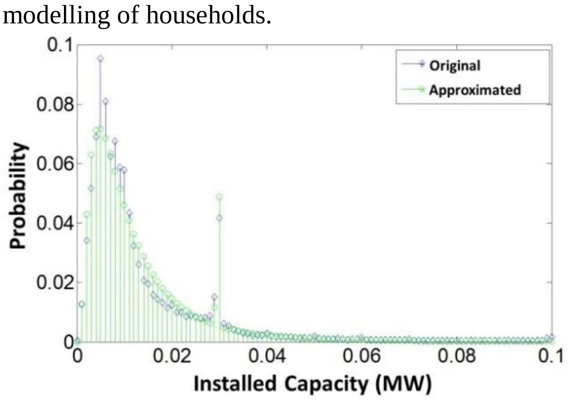


Figure 3: PPD of small PV units in Germany

Figure 4Figure 4 (top) shows the NPP of PV units in the course of a day during summer. The values are based on the direct and indirect solar radiation in TRY region 6 [8] and a PV model which considers typical orientation, ambient temperatures and typical module and inverter characteristics. It can be seen that the peak power of 1 p.u. is only fed in during noon with a low probability.

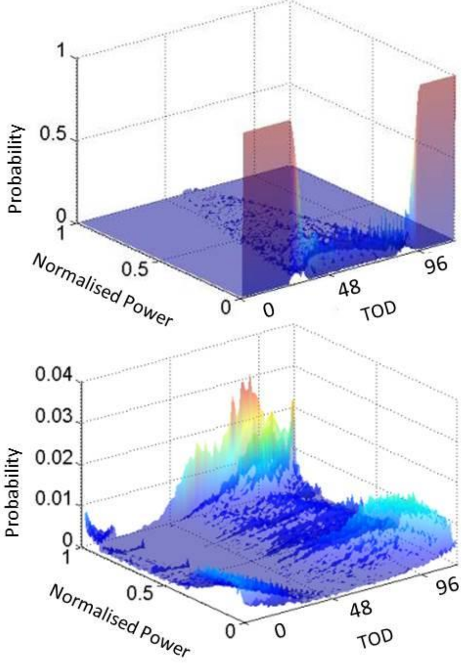


Figure 4: NPP of PV units during summer in TRY Region 6 (top) and Normalized charging power while charging for EV with CP=1 and CI=1 (bottom)

Electric vehicles (EV)

In [10-12] the charging processes of electric vehicles have been described with the parameters charging power (CP) and charging infrastructure (CI) according to Table 2. The probability that an electric vehicle (EV) is charging for all the CP-CI-combinations can be seen in Figure 5 for all TOD of a workday. With higher CP the charging probability of an EV decreases. At most CI scenarios the charging probability reaches its maximum in the evening.

Due to battery specific characteristics an EV does not charge with the maximum CP during the charging process. For CP=1 and CI = 1 Figure 4 (bottom) shows the NPP of an EV. In the morning the battery of the EVs is almost completely charged (and charging with small power) while during the day and in the evening it is emptier (and charging with higher power). The behaviour of the charging power cannot be expressed with lognormal functions like households, but requires more complex density functions, which cannot be expressed in simple analytical functions anymore. It depends on the capacity and charging characteristics of the batteries and the driving behaviour and routes of the users.

Charging Power (CP)	1	3.7 kW (AC, 1 phase, 16 A)
	2	11 kW (AC, 3 phase, 16 A)
	3	55 kW (DC, fast charge)
	4	70% of (1), 20% of (2), 10% of (3).
Charging Infrastruc-ture (CI)	1	At home
	2	At home and at work
	3	Everywhere
	4	At home and at work (outside the grid)

Table 2: Considered charging parameters

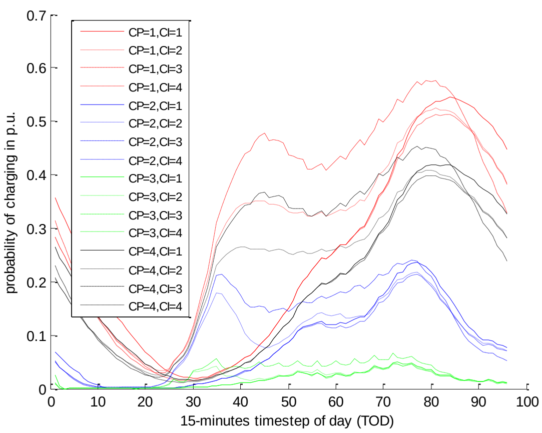


Figure 5: Probability of charging for CP-CI-combinations

CONVOLUTION METHOD

The vector of complex powers $\underline{\vec{S}}$ at the buses of a power system depend on the complex bus voltages $\underline{\vec{V}}$ and the well-known complex admittance matrix $\underline{\boldsymbol{Y}}$:

$$\underline{\vec{S}}\left(\underline{\vec{V}}\right) = \underline{\vec{V}}\,\underline{\vec{I}}^{*} = \underline{\vec{V}}\left(\underline{\boldsymbol{Y}}\,\underline{\vec{V}}\right)^{*}$$

This set of nonlinear equations can be linearized at an operating point (OP) resulting in the complex Jacoby matrix $\underline{\boldsymbol{J}}$ and the sensitivity matrices $\underline{\boldsymbol{A}}$ and $\underline{\boldsymbol{L}}$:

$$\underline{\boldsymbol{J}} = \left.\frac{\partial \underline{\vec{S}}(\underline{\vec{V}})}{\partial \underline{\vec{V}}}\right|_{\underline{\vec{V}}=\underline{\vec{V}}_{OP}} = \underline{\boldsymbol{A}}^{-1}$$

$$\underline{\boldsymbol{L}} = \left.\frac{\partial \underline{\vec{S}}_{ik}(\underline{\vec{S}})}{\partial \underline{\vec{S}}}\right|_{\underline{\vec{S}}=\underline{\vec{S}}_{OP}} = \left.\frac{\partial \underline{\vec{S}}_{ik}(\underline{\vec{V}})}{\partial \underline{\vec{V}}}\right|_{\underline{\vec{V}}=\underline{\vec{V}}_{OP}} \underline{\boldsymbol{A}}\,.$$

The voltages $\underline{\vec{V}}$ and power flows $\underline{\vec{S}}_{ik}$ (from bus i to k) are a simple addition of the respective values in the operating point and the multiplication of the sensitivity matrices with the deviations of input powers $\underline{\vec{S}}$ from the operating point:

$$\underline{\vec{V}} = \underline{\vec{V}}_{OP} + \underline{\boldsymbol{A}}\left(\underline{\vec{S}} - \underline{\vec{S}}_{OP}\right)$$

$$\underline{\vec{S}}_{ik} = \underline{\vec{S}}_{ik,OP} + \underline{\boldsymbol{L}}\left(\underline{\vec{S}} - \underline{\vec{S}}_{OP}\right)$$

When the input powers $\underline{\vec{S}}$ are regarded as probability density functions, their multiplication with the sensitivity matrices become a convolution process with the parameters of the sensitivity matrices as weighting factors.

Multilinearization

The linearization error increases for input powers $\underline{\vec{S}}$ which are far from the operating point $\underline{\vec{S}}_{OP}$. By choosing several operating points n_{OP} one can minimize the linearization error especially at the tails of a distribution by choosing several linearization points [1-2]:

$$\underline{\vec{S}}_{OP,k,max} = \underline{\vec{S}}_{EV}\,\left(1 - \beta_{k,max}\right) + \beta_{k,max}\underline{\vec{S}}_{max}$$

$$\underline{\vec{S}}_{OP,k,min} = \underline{\vec{S}}_{EV}\,\left(1 - \beta_{k,min}\right) + \beta_{k,min}\underline{\vec{S}}_{min}$$

One operating point is always set in the expected value of the input variables ($\beta_{1,min} = \beta_{1,max} = 0$) whereas two operating points can bet set to $\beta_{2,min} = 1 - \beta_{2,max} = 0.75$.

Correlations of input variables

Convolution operations are only applicable to uncorrelated input variables. For distribution grids PV units can be regarded as strongly correlated since they expect almost identical solar radiation. In the considered method input powers of PV units are set to separate busses. Prior to the convolution operation of the input variables of the entire distribution grid, these busses can be pooled with a correlation of one [3-4]. The pool of PV units can be regarded as uncorrelated with the rest of the distribution grid and the convolution process can be performed as before.

Test cases

For an analysis of the proposed convolution method the 8 bus test grid with 3 households in Figure 6 with the parameters listed in Table 3 has been used. In the following additional loads and generators are added to the grid according to Table 4 with a penetration rate of 50%. This means that the probability that a household has a PV unit or an EV is 50%.

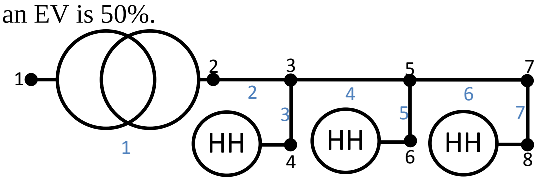


Figure 6: Considered test grid

Grid	R	X
Branch 1	36.75 Ω	93 Ω
Branch 2/4/6	100 $m\Omega$	50 $m\Omega$
Branch 3/5/7	12 $m\Omega$	2 $m\Omega$

Table 3: Test grid parameters

Input power types	AED	Peak power	Penetration Rate
HH	5 MWh	-	-
PV	-	10 kW	50 %
EV	-	11 kW	50 %

Table 4: Considered loads and generators

RESULTS

In this section the test cases according to Table 5 are discussed for a TOD of 60 and the summer case (see section "MODELLING OF DER") and have been implemented in MATLAB using [9].

Test case	Considered Power units	Regarded output	correlations
1	HH	Voltage @ bus 8	No
2	HH+PV	Voltage @ bus 8	No
3	HH+PV+EV	Voltage @ bus 8	No
4	HH+PV+EV	Voltage @ bus 11[1]	Yes
5	HH+PV+EV	Power @ branch 2	Yes

Table 5: Considered test cases

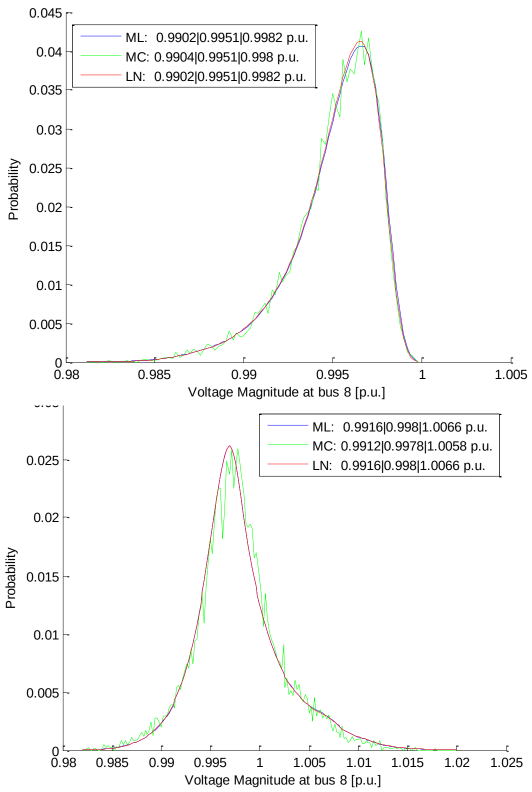


Figure 7: Results of test cases 1 (top) and 2 (bottom)

Figure 7 (top) shows the results according to test case 1. Next to the legend are given the 5%-Quantile, mean value and 95%-quantile of the resulting voltage probability density function. The distribution of the linearized (LN) and multilinearized (ML) voltages are almost identical with the results of the reference method (Monte Carlo (MC) simulation with 10.000 repetitions) and so are the mean values and defined quantiles. With PV units (test case 2) especially the mean value and 95%-quantile are increased significantly (see Figure 7 bottom).
With additional EV (test case 3) especially the 5%-quantile of the voltage distribution decreases to 0.975 p.u., whereas the 95%-Quantil decreases only slightly (see Figure 8 top). With a correlation of 1 between the PV units (test case 4) the probability of occurrence at the tails of the distribution increases whereas it decreases around the mean value of the distribution (see Figure 8 (middle)).

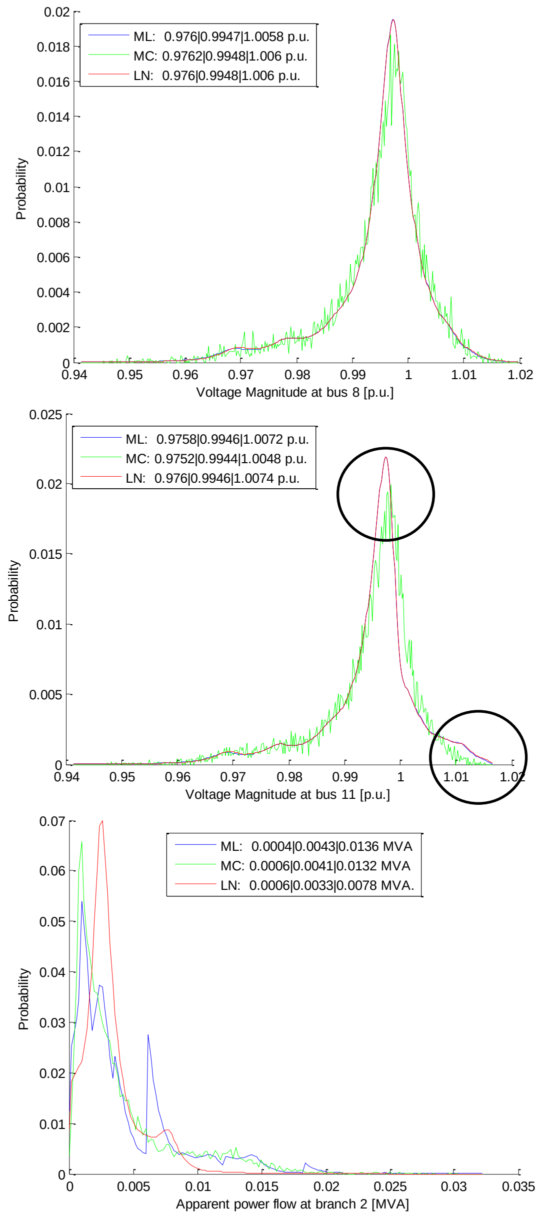


Figure 8: Results of test cases 3 (top), 4 (middle) and 5 (bottom)

Figure 8 (bottom) shows the apparent power of branch 2 according to test case 5. The linearized solution underestimates the probability of the distribution tails, where the result is falsified by the linearization error. Mean value and 95%-quantile are much better fitted with the multilinearized solution, which estimates the 95%-quantile to 13.2 kVAr (MC result is 13.6 kVAr).
The main advantages of the developed convolution

[1]Note that the correlations between the PV units require 3 additional busses, so that bus 8 becomes number 11 in this test case.

methods with respect to conventional Monte Carlo methods are:

- Reduction of calculation time by factor 8 for cases with low penetration rates and when only a few parameters of distribution grids are relevant
- Reduction of random access memory usage by minimum factor 40
- Analytical conceivability of input and output distributions.

Combined, these three advantages can significantly reduce the complexity, calculation time and random access memory requirements of studies like [13] significantly.

OUTLOOK

The proposed method has been expanded with the description of heat pump units and combined heat and power units in a probabilistic way, which could not be presented here.

Since the proposed method is implemented in MATLAB, probability density functions can internally only be modelled as discrete density functions. The resulting discretization and scaling errors and the (multi-) linearization errors still have to be investigated with conventional Monte Carlo simulations for each test case. In the future methods have to be developed, which clarify ex ante the necessary discretization of the input factors and the number of operating points in order to guarantee a certain maximum error with regard to bus voltages and power flows. This is topic to current research activities.

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