# STATE ESTIMATION OF LOW VOLTAGE MICROGRID USING AN ITERATIVELY REWEIGHTED LEAST SQUARES METHOD

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## ABSTRACT

*Higher penetration of small distributed energy resources will impact the low voltage network. As a consequence, better knowledge of real-time operating states of a distribution network is required. The use of an iteratively reweighted least squares state estimation method with real-time and pseudo-measurements obtained from a low-voltage microgrid is investigated. Simulation results for different types of measurements, different measurement configurations, and different time resolutions are reported. Simulation results show that the state estimator can be used effectively to extend the observability of the low voltage microgrid and provide reliable state estimates using different sets of measurements.*

## INTRODUCTION

Higher penetration of small-scale distributed generation is anticipated in many countries. As more generators are connected to distribution networks at the lower voltage levels, operational difficulties can arise. Therefore, better knowledge of real-time operating states of a distribution network is required. The lack of sensors and communication systems renders real-time monitoring and control of low voltage (LV) networks very limited and difficult. LV networks are under-determined systems with an inadequate number of real-time measurements to make the networks fully observable. Installing real-time measurements at all network nodes is impossible due to economic limitations. In the presence of a minimum set of real-time measurements, state estimation is used to extend the observability and to identify the operating states of a distribution network [1]–[4].

State estimation is defined as the computation of the minimum set of necessary values to try describe completely all other pertinent variables in a given system from some measurement data [5]. As a mathematical tool, state estimation acts as a noise filter to reduce the negative impact of errors in data. In the context of power systems, the state estimator acts as a filter between the raw measurements (e.g. voltage magnitude, active/reactive power injections, and active/reactive power flows) received from the system and all application functions that require the most reliable database for the current state of the system. In this sense, the state estimator minimises the error between real-time measurements and the calculated values of these measurements [3], [6], [7].

Distribution networks will become more meshed as larger numbers of distributed generators are connected. Hence, the use of state estimation methods that have been specially designed for radial networks would not be possible [2]. A state estimation algorithm based on the Iteratively Re-Weighted Least Squares (IRWLS) state estimation method used in transmission networks is presented. The performance of the IRWLS is evaluated with real-time measurements and pseudo-measurements obtained from a practical test system represented by the Distributed Energy Resources Test Facility (DER-TF) in Ricerca sul Sistema Energetico (RSE) in Italy. Different configurations of measurement devices and different time resolutions were used to assess the performance of the IRWLS state estimator with regard to the provision of reliable state estimates.

## DISTRIBUTION NETWORK STATE ESTIMATION

Distribution network state estimation is generally based on the classical transmission system state estimation. Weighted Least Squares (WLS) algorithms are used to solve the state estimation problem [6]. The WLS minimizes the sum of weighted squared residuals between the measured and estimated values as given by (1) subject to the constraints imposed by the measurement equations in (2)

$$\min\,(\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x}))^T\overline{\boldsymbol{W}}(\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x})) \quad (1)$$

$$\text{subject to } \boldsymbol{r}=\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x}) \quad (2)$$

where $\boldsymbol{z}$ is the measurements vector, $\boldsymbol{x}$ is the system state vector (voltage magnitude and voltage phase angle at different nodes of the network), $\boldsymbol{h}(\boldsymbol{x})$ is the vector of non-linear functions relating measurements to system state variables, $\boldsymbol{r}$ is the measurement residual vector, and $\overline{\boldsymbol{W}}$ is the weight matrix.

The weight matrix is a diagonal matrix where the weights of measurements are in the main diagonal. The weight assigned to each measurement reflects its accuracy. The accuracy of a measurement is expressed in terms of its variance ($\sigma^2$). Hence, the reciprocals of measurement variances ($1/\sigma^2$) are used as weights so that the final solution of the state estimator will be less impacted upon by measurements with higher variance than by measurements with lower variance.

In distribution networks, limited real-time measurements necessitate the use of pseudo-measurements in conjunction with the state estimator. Pseudo-measurements usually contain larger errors than real-time measurements. Real-time measurements are also susceptible to gross errors due to noises inherent in the power system and failure of measurement and communication devices. Structure of the measurement equations, location of the metering devices and network parameters are all factors that may lead to the creation of leverage points which are able to reduce the accuracy of the estimates. Generally, IRWLS algorithms are considered more robust than the WLS algorithms [8]–[11]. An IRWLS state estimator is used in this study. In IRWLS algorithms, $\overline{\boldsymbol{W}}$ includes functions that calculate measurement residuals at each iteration of state estimation. As a result, measurements are reweighted iteratively depending on the values of their residuals.

In the current study, voltage magnitude ($V_i$) and voltage phase angle ($\theta_i$) at each node ($i$) of the network[1] are taken as state variables. Therefore $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{\theta} \end{bmatrix}$.

The set of measurements available for this study comprises active/reactive power injection measurements ($P_i, Q_i$) and voltage magnitude measurement $V_i$ at each node ($i$) of the network, and active/reactive branch power flow measurements ($PF_{ij}, QF_{ij}$) between nodes $i$ and $j$ of the network. Therefore $\boldsymbol{z}^T = [\boldsymbol{P}\ \boldsymbol{Q}\ \boldsymbol{PF}\ \boldsymbol{QF}\ \boldsymbol{V}\ ]$.

For a network with a number of $N$ nodes, the power flow measurements are represented as

$$PF_{ij} = g_{ij}V_i^2 - V_iV_jg_{ij}\cos\theta_{ij} - V_iV_jb_{ij}\sin\theta_{ij} \quad (3)$$

$$QF_{ij} = -b_{ij}V_i^2 - V_iV_jg_{ij}\sin\theta_{ij} + V_iV_jb_{ij}\cos\theta_{ij} \quad (4)$$

where $g_{ij}$ and $b_{ij}$ are the conductance and susceptance of the branch connecting nodes $i$ and $j$ of the network respectively [6]. The active/reactive power injections at each node ($i$) of the network are

$$P_i = \sum_{j=1}^{N} PF_{ij} \quad (5)$$

$$Q_i = \sum_{j=1}^{N} QF_{ij} \quad (6)$$

In eqns. (3) through (6), $j = 1, 2, \ldots, N$ and $j \neq i$ [9].

The vector $\boldsymbol{h}(\boldsymbol{x})$ of non-linear equations comprises the partial derivatives of measurement functions with respect to state variables (voltage magnitude and voltage phase angle).

## TEST MICROGRID

The Distributed Energy Resources Test Facility (DER-TF) is a three-phase LV Microgrid [13] consisting of several generators with different technologies (renewable and conventional), controllable loads and storage systems. The single line diagram of the DER-TF network configuration is shown in Fig.1. The microgrid is connected to the distribution grid through a 23kV/0.4kV transformer. In Fig. 1, Bus 1 is the grid connection point. The measurement interface –Bus 2– is the network node where real-time measurements of voltage magnitude, power injections and power flows are obtained. Several tests with different network configurations have been carried out. The main aims of these tests were to validate the performance of the state estimator in a practical LV network and to investigate the impact of both measurement configurations and time granularity on the output of the state estimator.

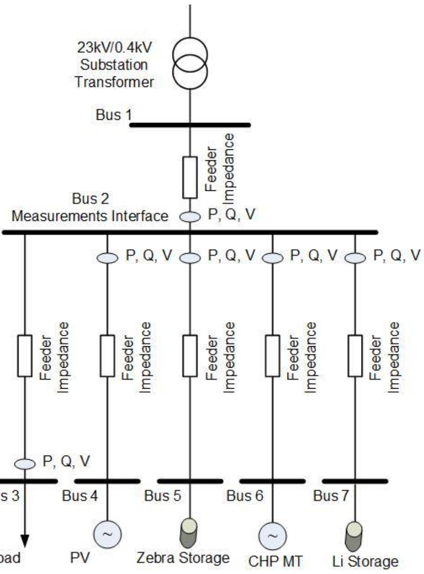


**Fig. 1. Single line diagram of the DER-TF**

The distributed energy resources available for the test were a photovoltaic (PV) field, a Zebra battery storage system, a Gas Combined Heat and Power (CHP) micro turbine, a Lithium battery storage system, and a programmable resistive and inductive load.

## SITE TEST AND SIMULATION RESULTS

Taking into consideration both network topology and available distributed energy resources connections, different measurement configurations were used to validate the performance of the state estimator. Initially, the state estimator was run with a measurements set comprising real-time voltage magnitudes, active/reactive power injections, and active/reactive branch power flows. Real-time voltage measurements, active/reactive branch power flows and active/reactive power injections were separately input to the IRWLS in order to assess the impact of different types of measurements on state estimates. The influence of changing measurements' time resolution on state estimates was also simulated.

The network configuration used for testing state estimator performance is depicted in Fig. 1. Measurements were acquired at 2 seconds rate and saved at 10 seconds rate (10 seconds average values) in binary form. In order to make measurements accessible and readable by spreadsheet software (like Microsoft Excel) or other programs (e.g. IRWLS state estimator), RSE developed a software platform that was used to extract the measurements and save them in text mode. This software platform is able to calculate the average values of measurements based on user defined periods. The data extracted from this software is used as an input to the state estimator.

1 Except voltage phase angle at the grid connection point (Bus 1)

## Simulation results considering an over-determined system of real-time measurements

At first, a set of real-time measurements comprising voltage magnitude and power injections at Bus 2 (Measurements interface) and Bus 3 (Load Bus); and branch power flows at all feeders of the network was input to the state estimator. Voltage magnitudes at Bus 1 through Bus 7 and voltage phase angles at Bus 2 through Bus 7 represent the output of the state estimator. The set of measurements at the infeed of Bus 2 was used to calculate the voltage magnitude and active/reactive power injections at Bus 1 (the grid connection point – GCP) because of the non-availability of real-time measurements at that bus (the same procedure has been applied to Bus 4 through Bus 7). A comparison between the estimated voltage and the calculated voltage at Bus 1 is shown in Fig. 2. It is clear that the state estimator provided reliable estimates of the voltage magnitude even when there was a gross error in the real-time measured voltage at infeed of Bus 2 (where measured voltage was 246.374 volts at around 12:09pm).

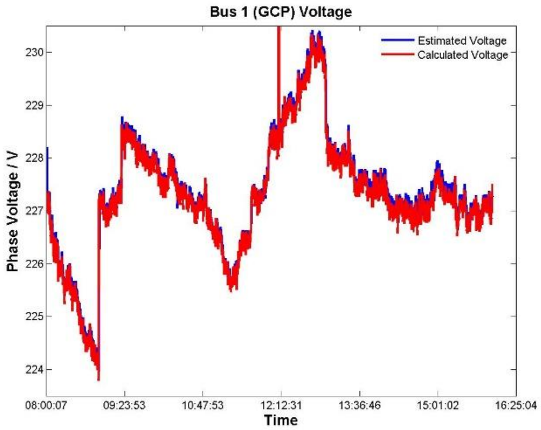


**Fig. 2. Comparison of Bus 1 (GCP) voltages using 10-seconds interval**

The presence of a large number of gross errors within measurements is known to disable conventional WLS state estimators [4],[9]. However, results showed that even under such a condition of extreme gross errors, the IRWLS state estimator still provided reliable results of network states. Fig. 3 presents a comparison between the measured and the estimated Bus 3 voltages using a 10-second interval. The results depicted in Fig. 3 show a clear offset between the estimated and the measured voltages. This offset indicated a possibility of bias and inherent offset within Bus 3 measurement devices. An investigation test was carried out under no-load conditions to compare the voltages at Bus 2 and Bus 3. The no-load test confirmed that the measurement devices at Bus 3 contained an offset of less than 1 volt. A recommendation was made to update the measurement devices at Bus 3. This investigation demonstrates the effectiveness of the IRWLS estimator.

The output of the state estimator not only includes network states in terms of voltage magnitude and voltage phase angle at each node in the network, but also includes calculated active/reactive power injections at each node

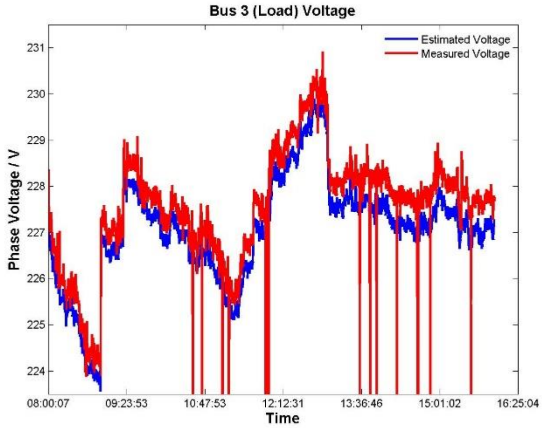


**Fig. 3. Comparison of Bus 3 (Load) voltages using 10-seconds interval**

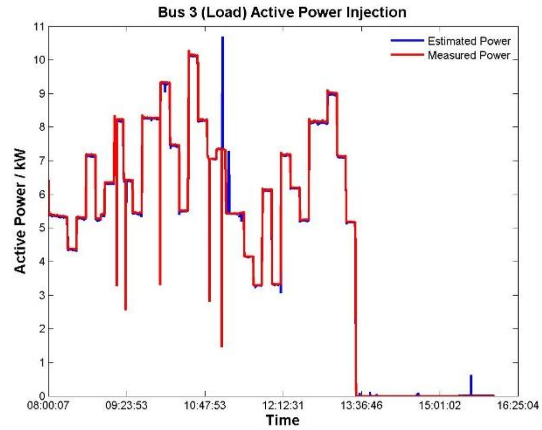


**Fig. 4. Comparison of Bus 3 (Load) active powers using 10-seconds interval**

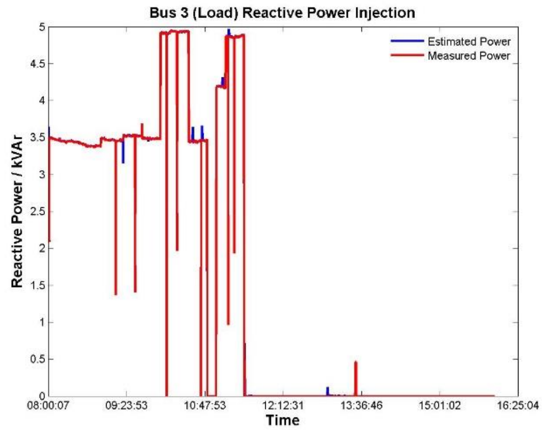


**Fig. 5. Comparison of Bus 3 (Load) active powers using 10-seconds interval**

based on the estimated network states. As a result, it is possible to compare real-time power injections with estimated ones. Fig. 4 shows a comparison between the measured and estimated active power injections at Bus 3. The real-time measured reactive power injection as compared to the estimated reactive power injection is depicted in Fig. 5.

## Impact of real-time voltage measurements

The impact of voltage measurements on the IRWLS state estimator was investigated. An assumption was made that real-time voltage measurements exist at Bus 2 and Bus 3. Given that power injections and power flows are often measured in real-time at distribution network substations, power injections at infeed of Bus 2 (representing GCP power injections plus power loss along the main feeder) and power flows along the main feeder were assumed to exist in real-time. It was also assumed that pseudo-measurements represent power injections at all network nodes other than the infeed of Bus 2 and that no power flow measurements elsewhere exist. Results show that the loss of both real-time power flow measurements and real-time power injections does not impact the estimated voltages. As an example, the calculated versus the estimated voltages are depicted in Fig. 6. It is obvious that Fig. 6 exactly matches Fig. 2 implying that voltage estimates are mainly influenced by the presence of real-time voltage measurements.

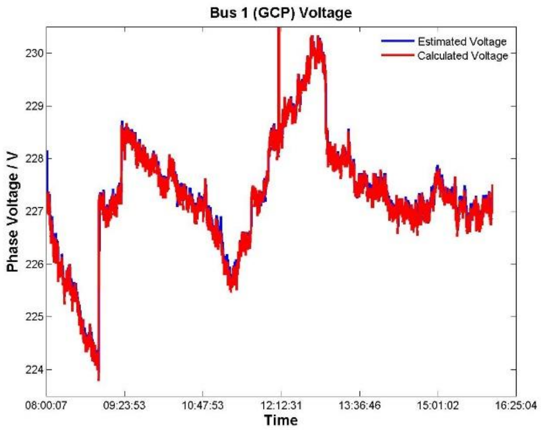


**Fig. 6. Bus 1 (GCP) voltages using 10-seconds interval with real-time voltage measurements**

## Impact of real-time branch power flow measurements

Power flow measurements' impact on state estimates was taken into consideration. The set of real-time active/reactive branch power flow measurements along all network feeders was made available to the state estimator. Real-time voltage and power injection measurements at infeed of Bus 2 were also fed to the state estimator. At other network nodes, it was assumed that no voltage measurements exist and that power injections are represented as pseudo-measurements. Fig. 7 illustrates a comparison between real-time measured and estimated voltages at Bus 3. It is obvious that reliable voltage estimates were still provided by the state estimator even in the presence of a minimum set of real-time voltage measurements (voltage measurement at infeed of Bus 2). However, it is also clear that Bus 3 voltage estimates were impacted upon by the presence of voltage measurement gross error (voltage measurement at 12:09pm). This confirms the correlation between real-

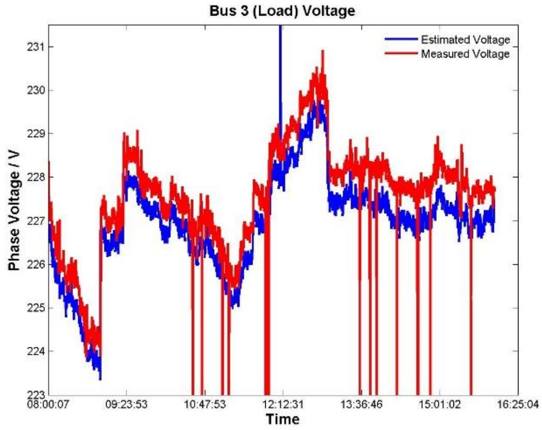


**Fig. 7. Bus 3 (Load) voltages using 10-seconds interval with real-time power flow measurements**

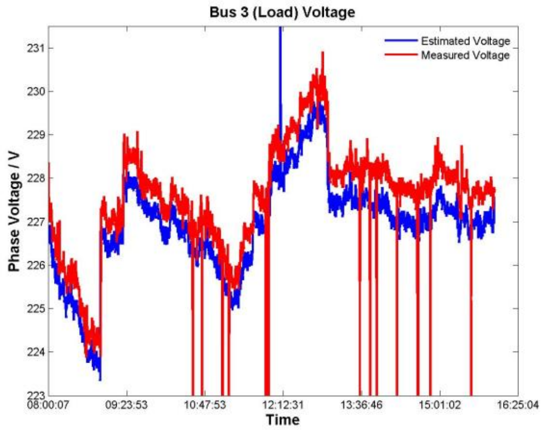


**Fig. 8. Bus 3 (Load) voltages using 10-seconds interval with real-time power injection measurements**

time voltage measurements and voltage estimates.

## Impact of real-time power injection measurements

The impact of real-time power injection measurements was also studied. Real-time measurement set of power injections at Bus 2 and Bus 3, voltage magnitude (at infeed of Bus 2) and power flows (along the main feeder) were used by the state estimator. A comparison between the measured and estimated voltages of the load is shown in Fig. 8. It is clearly shown that Bus 3 (Load) voltage was reliably estimated. The existence of a gross error in real-time measured voltage at the infeed of Bus 2 emphasizes the fact that the accuracy of voltage measurements is the most important factor for accurate voltage estimates.

## Impact of time granularity:10-mins. granularity

Tests carried out in the previous sections used measurements obtained at 10-seconds intervals. The performed tests gave a detailed insight into the network and how accurate state estimates were obtained in the presence of gross errors within different real-time measurements and with different types and configurations of the set of measurements available to the state estimator. In order to demonstrate the impact of changing time granularity on the state estimator performance, the 10-minutes average values of measurements were retrieved from the RSE software platform. The basic assumption of a complete set of real-time measurements available as state estimator inputs is made. Fig. 9 shows a comparison between real-time measured and estimated voltage at Bus 3.

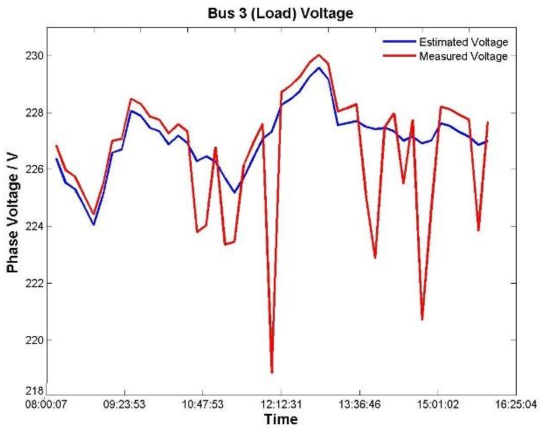


**Fig. 9. Bus 3 (Load) voltages using 10-minutes interval with real-time power flow measurements**

It is clear that the state estimator produces reliable outputs even when the average values of measurements are obtained at time rates higher than the original 10-seconds rate. Gross measurements errors were properly defined and filtered as shown in Fig. 9.

## CONCLUSIONS

Real-time measurements obtained from a practical low-voltage network were used to test and validate the performance of an IRWLS state estimator. Different measurement types and configurations, and different time resolutions were simulated. Reliable output of the state estimator in the presence of gross measurements and with the utilisation of different types of measurements demonstrate the versatility of the IRWLS estimator. The results obtained show that the IRWLS state estimator can be integrated with distribution networks (both LV and MV) to increase network observability and provide real-time network state in terms of the voltage magnitudes and voltage phase angles at all nodes in the network.

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