STOCHASTIC DYNAMIC PROGRAMMING APPROACH FOR PROACTIVE REPLACEMENT OF POWER CABLES

Swati SACHAN, Chengke ZHOU, Geraint BEVAN and Babakalli ALKALI
Glasgow Caledonian University – UK
Swati.Sachan@gcu.ac.uk; C.Zhou@gcu.ac.uk; Geraint.Bevan@gcu.ac.uk; Babakalli.Alkali@gcu.ac.uk

ABSTRACT

The purpose of this paper is to present a stochastic dynamic programming based model to solve the optimization problem of cable replacement. The proposed methodology can be implemented on cables with known failure distribution and insulation degradation level; the methodology to estimate both of the elements is based on previously developed Non-homogenous Poisson Process model (NHPP) and stochastic degradation model, respectively. The model gives the sequence of decisions for each year of the planning horizon such that it optimizes the overall cost and improves the reliability by lowering the frequency of unplanned outage. The model was tested on an unjacketed XLPE cable.

INTRODUCTION

In recent years many methods have been proposed and utilized for the maintenance and replacement of engineering assets, of which dynamic programming is the most widely used [1,2,3,4]. The dynamic programming approach can estimate the optimal cost effective decision policy for assets which are required to operate indefinitely. The effective decision policy may include decisions for preventive and corrective maintenance. Preventive maintenance improves the reliability by preventing failure causes; corrective maintenance restores the cable to its operational state after the occurrence of a failure. A preventable and correctable failure model provides an extended planning horizon for making a strategic plan.

In this paper the length of planning horizon is estimated by a degradation model which enables prediction of the evolution of cable insulation condition. A stochastic dynamic programming model is introduced which considers the fact that cable failure has a certain degree of random nature and the problem of optimal planning can be solved mathematically by considering failure as a stochastic process. The model gives the sequence of decisions for each year of the planning horizon such that it optimizes the overall cost and improves the reliability by lowering the frequency of unplanned outage or SAIFI (system average interruption frequency index). The decision space consists of four kinds of decisions, “keep (K)”, “preventive maintenance (PM)”, “corrective maintenance (CM)” and “replace (RP)”. Preventive maintenance impacts the frequency of unplanned outages by preventing the cause of failures.

FINITE PLANNING HORIZON

A finite planning horizon can be determined by a previously developed stochastic degradation model [6]. The model probabilistically estimates the degraded state of insulation with the advancement of age. The degradation process of all types of cables varies with the cable material and manufacturing process. The degradation level and planning horizon a_0 to a_J of a cable population installed in consecutive years I_0 to I_J is shown in Fig 1. These cables have similar design and operational conditions. The degradation remains negligible for a long period of time before it worsens dramatically. A levcl of 75% can be considered as the maximum acceptable degradation condition of the cable.

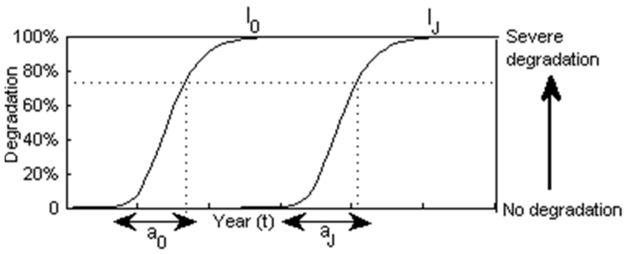

Fig 1: Degradation of cable insulation with respect to service life

Rational information on cable condition is required to justify investment decisions such as proactive replacement. Power cable failure occurs due to random, ageing or a combination effect of both causes. Random failures create fluctuations in the historical failure rate data. These fluctuations in estimated failure rate should not drive the proactive cable replacements. A random failure can occur due to degradation in a small section of a cable circuit whereas ageing failures occur due to slow and continuous degradation of the entire cable insulation due predominant effect of electro-thermal stress in the daily load cycle [5,6]. Faults or failures due to random causes can be rectified by cutting and splicing new cable parts into the small affected sections of the cable. Therefore, the best optimal replacement decision would be to replace the cable when its entire insulation is in poor condition or when the overall maintenance (PM and CM) and failure costs outweigh the replacement costs before the end of finite planning horizon.

COST

The optimal decision policy depends on four types of costs: replacement cost (C_{RP}), failure cost (C_F) , maintenance cost (C_M) and repair cost (C_R) [7,8]. Current preventive maintenance practices and technology are not capable of detecting all failure causes. Therefore, there are two possible kinds of repair. First, repair when the potential failure causes are detected by PM. Second, repair when corrective maintenance (CM) is carried out on failed cable, when the failure cause remains undetected and the cable eventually fails in the future. The PM repair cost ($C_{R_{PM}}$) is generally less than CM repair cost ($C_{R_{CM}}$).

STOCHASTIC DYNAMIC PROGRAMMING

The planning horizon is from $t = 0$ to $t = T$, as shown in Fig 2. The failure distribution of power cables is obtained from the power-law NHPP model; it considers the fact that cable section is a repairable component. Its detailed application in power cables is shown in [5]. Let the failure distribution function of similar cables (similar in terms of design and installation year) be $F(a) = P(t \leq a)$ where, t is the failure time and a is the age of the cables. The first extended curve in Fig 2 shows the failure distribution of these cables under no maintenance or with unknown past maintenance information. The PM action reduces the failure probability, however, the PM can only detect some potential failure causes and other causes remain undetected. The effect of applying PM reduce the failures with $x\%$ [8]. It is assumed that the failure probability of the cables is reduced by the same percentage and this affects the age of cable in comparison to cables without maintenance. The reduced failure probability is:

$$p(a') = p(a)[1 - \sum_{z=1}^{\mathbb{Z}} x_z \%] \tag{1}$$

where, a and a' are chronological age and effective age respectively. The effective age shows the impact of maintenance and it is associated with the failure probability. If the failure probability of a cable under maintenance is less than cable under no maintenance, then maintenance has a positive impact on the condition of the cable and $a' < a$. Similarly, if the failure probabilities are same then, maintenance has no effect on cable condition and $a' = a$.

Stage and State

It is assumed that one optimal decision is taken at the beginning of the year. Therefore, each year (t) is a stage,

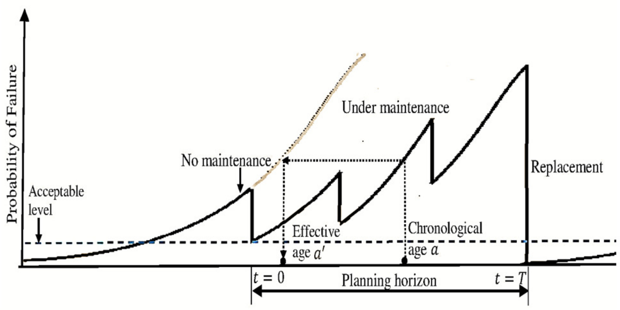

Fig 2: Planning horizon and effective age after preventive maintenance

where $t = 0$ *to* T. At any stage t cable can only be in two states, either it will be in operating state with an effective age of a'_t or in failed state $F_{a'_t}$, here $F_{a'_t}$ is the failure at an effective age a' at stage t. Set of states S: $\{a'_t, F_{a'_t}\}$

Decision

Four types of decisions can be taken at any stage t. First, “keep (K)” a cable without taking any action. Second, “preventive maintenance (PM)” which reduce the risk of failure or in other words failure frequency. Third, “corrective maintenance (CM)” if the cable fails due to lack of maintenance or due to failed maintenance procedure. Fourth, “replace (RP)” for the replacement of old cable with the new cable. Set of decisions D: $\{K, PM, CM, RP\}$

Table 1: Decision space for all states

	State S	
	a'_t	$F_{a'_t}$
Decision D	K, PM, RP	CM

State transition probability

A cable transits from one state to another state when a decision D is taken. The probability that a cable transits from a state at stage t to another state at stage $t+1$ depends on current state and the decision taken at that state. If a cable is in operating state a'_t, then, three kinds of decision $D = \{K, PM, RP\}$ can be taken. By taking these decisions, cable can transit either to another operating state $(\bar{F})$ or it can transit to a failed state (F). Suppose at any stage t of the planning horizon, the cable is at state a'_t. The keep (K) decision at this state will transit the cable condition to one of two possible states in the next stage $t + 1$. It can either transit to an operating state $a'_{t+1} = a'_t + 1$, where, cable ages by a year or it can transit to a failed state $F_{a'_{t+1}}$.

$$\text{K:} \begin{cases} F_K: P(F_{a'_{t+1}} | a'_t, K) = P(a'_{t+1} | a'_t, K) \\ \bar{F}_K: P(a'_{t+1} | a'_t, K) = 1 - P(a'_t + 1 | a'_t, K) \end{cases} \tag{2}$$

The preventive maintenance (PM) decision at state a'_t can detect $x\%$ of failures and reduce the failure probability by the same percentage. The undetected failure causes and few unsuccessful PM actions eventually transit the cable to the failure state in next stage $t + 1$, as shown in Fig 3. The transition probability for PM action is:

$$\text{PM:} \begin{cases} F_{PM}: P(F_{a'_{t+1}} | a'_t, PM) = P(UD) + P(D).P(USF) \\ \bar{F}_{PM}: P(a'_{t+1} | a'_t, PM) = P(D).P(SF) \end{cases} \tag{3}$$

The existing cable at state a'_t can also be replaced by a new cable. The new cable will have a different failure distribution than the old cable. At the next stage t+1 the new cable will have age 1. If we assume that installation practices are reasonably reliable then it will be have negligible failure probability at age 1 and it is highly likely that cable will transit to an operating state $a'_{t+1} = 1$, shown in equation (4).

$$\text{RP:}\begin{cases} F_{RP}: P(F_{a'_{t+1}} | a'_t, RP) \approx 0.01 \\ \bar{F}_{RP}: P(1 | a'_t, RP) = 1 - P(F_{a'_{t+1}} | a'_t, RP) \approx 0.99 \end{cases} \quad (4)$$

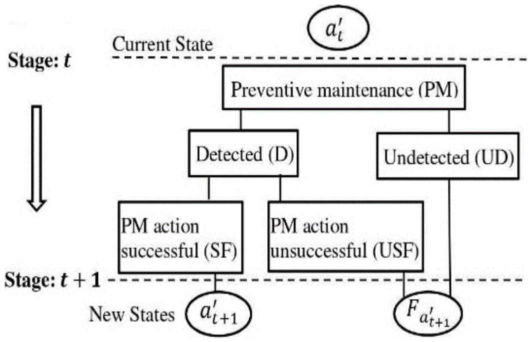

Fig 3: Preventive maintenance transition probability

If a cable is in failed state $F_{a'_t}$ then, only the decision $D = \{CM\}$ can be taken. By doing CM, the cable can regain its operating state ($\bar{F}$) or it can again land on a failed state (F). The CM could be perfect, minimal, imperfect and worst repair; which restore cable to an operating with "good as new", "bad as old", between "good as new" and "bad as old" and, failed state, respectively. Here, it is assumed that CM restores cable to a condition between "good as new" and "bad as old "conditions with F_{CM} probability.

$$\text{CM:}\begin{cases} F_{CM}: P(F_{a'_{t+1}} | F_{a'_t}, CM) = 1 - P(a'_t | F_{a'_t}, CM) \\ \bar{F}_{CM}: P(a'_{t+1} | F_{a'_t}, CM) = P(a'_t | F_{a'_t}, CM) \end{cases} \quad (5)$$

Objective function and recursive function

The objective in Equation (6) is to minimize the total cost of maintenance over finite planning horizon. It is achieved by recursively solving the set of Bellman equations for all the possible states the system might visit in future. The group of Equations (7) describe the costs associated with each decision: the keep (K) decision has no immediate cost whereas; preventive maintenance (PM), replacement (RP) and corrective maintenance (CM) have an immediate cost of repair and maintenance, replacement and, failure and repair cost, respectively. $V_{t+1}(.)$ is the expected future cost from transition state to the end of the planning horizon.

$$\text{Objective: } \min \sum_{t=0}^{T} C_F + C_{RP} + C_M + C_R \quad (6)$$

$$\min\begin{cases} \boldsymbol{K}: \bar{F}_K V_{t+1}(a'_{t+1}) + F_K V_{t+1}(F_{a'_{t+1}}) & (7) \\ \boldsymbol{PM}: C_M + C_{R_{PM}} + \bar{F}_{PM} V_{t+1}(a'_{t+1}) + F_{PM} V_{t+1}(F_{a'_{t+1}}) \\ \boldsymbol{RP}: C_{RP} + \bar{F}_{RP} V_{t+1}(1) + F_{RP} V_{t+1}(F_{a'_{t+1}}) \\ \boldsymbol{CM}: C_F + C_{R_{CM}} + \bar{F}_{CM} V_{t+1}(a'_t) + F_{CM} V_{t+1}(F_{a'_{t+1}}) \end{cases}$$

$0 < t < T$

TEST MODEL

The proposed methodology for maintenance can be implemented on cables with known failure distribution and extent of insulation degradation. The model was tested on an unjacketed XLPE cable installed in 1977. It is a lateral cable of length 500 m which distributes power to 42 households. The failure distribution and insulation degradation level are shown in Fig 4 and Fig 5, respectively, and the methodology to estimate both the elements and information the cable can be seen in [5,6].

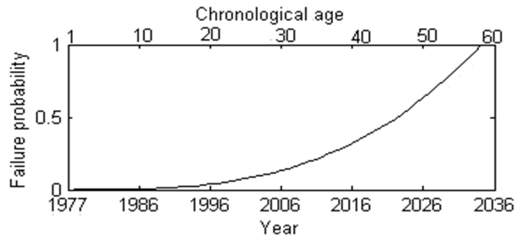

Fig 4: Failure distribution

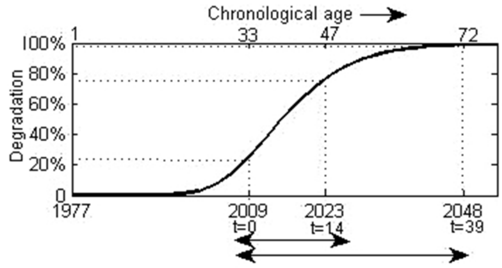

Fig 5: Insulation degradation level and planning horizons

It is assumed that the current year is 2009; and by year 2023 and 2048 the entire insulation of the cable is expected to degrade to 75% (moderately severe) and 99.8% (severe), respectively. The maintenance was planned in two time horizons, first from year 2009 - 2023 ($t = 0\ to\ 14$) and second from year 2009 - 2048 ($t = 0\ to\ 39$), shown in Fig 5. The PM action in the planning period reduces the random failures. The water treeing phenomena is one of the main causes of failure in unjacketed XLPE cables produced in mid-1970s. The reduction percentage can be estimated from past experience; here, it was assumed that the preventive maintenance (PM) can detect 65% (0.65) of failure causes and reduce the failure probability by the same percentage. The transition probability of PM action is $F_{PM} = 0.58$ and $\bar{F}_{PM} = 0.42$ (from Equation (3) $F_{PM} = 0.65 \times 0.90$ and $\bar{F}_{PM} = 0.35 + 0.65 \times 0.10$). The transition probability of keep (K) and corrective maintenance (CM) is obtained from the failure distribution as explained in the previous section. The failure probability of 0.08 (8%) is assumed as the minimum acceptable level. The PM and RP decisions are not taken below this level.

The input cost data in the model is shown in Table 2. It must be noted that, usually the cost of preventive maintenance (diagnostic tests and inspection) is negligible in comparison to repair, replacement and failure cost. The failure cost a cable depends on the consumption profile of the customers which has huge impact on result of the model. The failure cost in this case is low as, the lateral cable serves residential customers.

Table 2: Maintenance and Failure Cost

Cost	Value
1. Replacement cost: Let, the existing unjacketed XLPE cables will be replaced by TR-XLPE cable. **Replacement cost for 500 m cable (C_{RP}):**	£47572.18
2. Failure cost: The XLPE cable in this example distribute electricity to 42 residential customers (h). Average annual residential load: Average hourly power consumption (L_h): Average unplanned interruption time (t_r): Power outage cost (d_h): Time dependent power outage cost (b_h): **Average failure cost $C_F = (d_h + b_h t_r)L_h$:**	 $6000\ kWh$ $28.76\ kW$ $\left(\frac{6000kWh \times 42}{8760\ h}\right)$ $2.5\ h$ $1.2\ £/kW$ $5.0\ £/kWh$ £402.70
3. Maintenance cost: **Average maintenance cost for 500 m cable (C_M):**	£250.0 (500£/km)
4. Repair cost: The average repair cost of single failure is **Average CM repair ($C_{R_{CM}}$):** **Average PM repair cost ($C_{R_{PM}}$):**	 £4000.00 £500.00

The optimal policy which minimizes the cost over the planning horizon is shown in Fig 6. At the beginning of the planning horizon ($t = 0$, 2009) cable is in operating state with effective age $a' = 33$ (effective age is same as chronological age $a' = a$, because no maintenance action had been taken). In planning horizon $t = 0$ *to* 14, the model suggests preventive maintenance (PM) at two instances in the planning horizon; first at $t = 1$ (2010) and then in $t = 7$ (2016); and keep (K) decision in all other stages. It does not suggest replacement (RP) in this planning horizon as maintenance cost does not exceed the replacement cost due to the positive effect of PM and low failure cost of the cable.

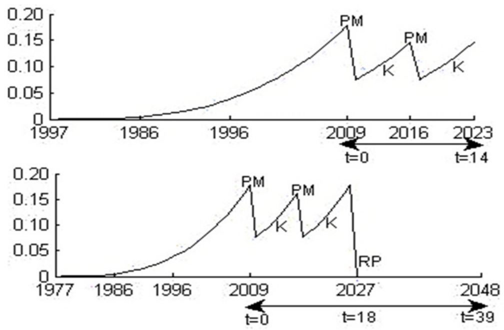

Fig 6: Optimal decision policy for $t = 0$ to 14 and $t = 0$ to 39 planning horizon

The lengthiest planning horizon when the cable insulation condition reaches 99.8% is from $t = 0$ *to* 39 (2009-2048). The model suggests PM at $t = 1$, $t = 8$ and replacement (RP) at $t = 18$ (2027) as the optimal decision policy which minimizes the cost over this planning period. The result shows that, by the implementation of preventive maintenance (PM) cable can be kept in service until $t = 14$ (2023) with minimum maintenance cost (a cost which does not exceed replacement cost) at moderately severe insulation condition. However, the cable must be replaced with TR-XLPE at or before $t = 18$ (2027) because at this year the cable maintenance cost exceeds replacement cost and the entire insulation is expected to have severe degradation. The severe degradation in the entire insulation and high maintenance cost compared to replacement cost is a justifiable reason to support the proactive replacement of the unjacketed cables between the years 2023 to 2027 ($t = 14$ to 18).

CONCLUSION

The proposed stochastic dynamic programming model is capable of finding the optimal decision policy with respect to optimal long run cost for a cable with a known failure distribution and degradation level. The optimal policy improves the reliability by suggesting the appropriate time for preventive maintenance and replacement action. The utilities and regulators can assess the monetary risks by exploiting the probabilistic nature of the model.

REFERENCES

[1] E. Abbasi, M. F. Firuzabad and A. A. Jahromi, 2009, "Risk based maintenance optimization of overhead distribution networks utilizing priority based dynamic programming ", *Power & Energy Society General Meeting IEEE*, 1-11.

[2] A. J. Bloom, A.; C. Feinstein and P. Morris, 2006, “Optimal Replacement of Underground Distribution Cables ", *Power Systems Conference and Exposition, IEEE PES*, 389-393.

[3] J. Korpijavi and J. Kortelainen, 2009, “A dynamic programming model for maintenance of electric distribution system ", *World Academy of Science, Engineering and Technology*, Vol.3, 636–639.

[4] K. S. Moghaddam and J. S. Usher, 2011, "Preventive maintenance and replacement scheduling for repairable and maintainable systems using dynamic programming", *Computers & Industrial Engineering,* vol. 60, Iss.4, 654-665.

[5] S. Sachan, C. Zhou, G. Bevan and B. Alkali, 2015, “Prediction of power cable failure rate based on failure history and operational conditions,” *9th International Conference on Insulated Power Cables.*

[6] S. Sachan, R. Wen, Y. Xiang, L. Yao and C. Zhou, 2015, “A stochastic electro-thermal degradation model of power cables” *High Voltage Engineering*, Vol. 41, Iss. 4, 1178-1187.

[7] J. Lassila, S. Honkapuro and J. Partanen, 2005, "Economic analysis of outage costs parameters and their implications on investment decisions ", *Power Engineering Society General Meeting,* Vol. 3, 2165-2172.

[8] L. Bertling, R. Allan and R. Eriksson, 2005, " A reliability-centered asset maintenance method for assessing the impact of maintenance in power distribution systems", *IEEE Transactions on Power Systems, ,* Vol. 20, Iss.1, 75-82.