

## REAL-TIME PRICING OF DISTRIBUTION NETWORK BASED ON NODE LOAD SENSITIVITY

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### ABSTRACT

The smart distribution network requires the customers to participate in the regulation of grid, optimizes the electric curve and realizes the peak load shifting by using the real-time pricing. According to the geographical distribution of loads in the distribution network, the sensitivity of the loss of distribution network to nodal load is derived, and the model of the sensitive relationship of load node is provided. Combing the model and the elasticity curve of electric demand, the influence of different kinds of loads on real-time pricing is analysed. And the model of real-time pricing for distribution network is proposed. The simulation result based on the IEEE69 nodes system shows that the longer power supply radius and the bigger the load density are, the bigger the sensitivity of nodal load is and the higher the real-time price of node is. At the same time, the bigger the elastic demand is, the more the real-time pricing changes. The result also shows that the peak-valley difference of the grid is reduced significantly.

### INTRODUCTION

<sup>1</sup>With the rapid growth of economy, the power demand and the peak-valley difference are increasing. It will be a substantial waste to meet increased peak load demands by expanding power grid. Demand response is a kind of behavior to shift the peak load and improve the electricity efficiency. Meanwhile, the real-time pricing mechanism is an effective way for demand response. The real-time pricing mechanism allows the consumers to choose the electricity-using mode according to their demands and the current power price. Then, it is very important to construct a reasonable real-time pricing mechanism for distribution network <sup>[1-2]</sup>.

In recent years, real-time pricing theory has been researched. However, most of the present researches are mainly focused on the transmission network <sup>[3]</sup>, and some applications of nodal pricing for distribution network <sup>[4]</sup>. Base on the radial structure of distribution network, the load sensitivity model is derived to reflect the impacting on the real-time price considering the load distribution in time and space. Based on the effect of price elasticity of

the demand side on different types of loads <sup>[5]</sup>, the demand price elasticity function are proposed to calculate the impact of different types of loads on the real-time price. And then an adaptive model of real-time price is established based on the above conditions.

### NODAL LOAD SENSITIVITY MODEL OF DISTRIBUTION NETWORK

The structure of distribution network is shown in Fig. 1.

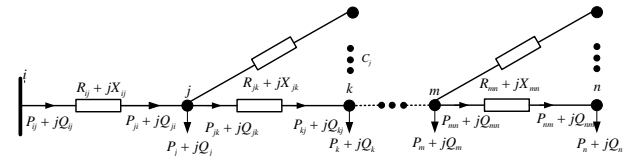


Fig.1 Distribution network in simplified structure

In branch  $i - j$ ,  $R_{ij} + jX_{ij}$  is the branch impedance,  $P_{ij} + jQ_{ij}$  is the head branch power,  $P_j + jQ_j$  is the end branch power.  $C_j$  is the child node set for node  $j$ .

The expression of the end branch power for branch  $i - j$  is:

$$P_{ji} = P_j + \sum_{k \in C_j} P_{jk} \quad (1)$$

$$Q_{ji} = Q_j + \sum_{k \in C_j} Q_{jk} \quad (2)$$

$n$  is a random node in the downstream of branch  $i - j$ . The partial derivative of total power loss with respect to the injection power of node  $n$  is:

$$\frac{\partial P_{loss}}{\partial P_n} = \frac{\partial P_{ij}}{\partial P_n} - 1 \quad (3)$$

$$\frac{\partial P_{ij}}{\partial P_n} = \frac{\partial P_{ij}}{\partial P_{ji}} \times \frac{\partial P_{ji}}{\partial P_n} \quad (4)$$

Where  $P_{loss}$  is the total active power loss;  $P_{ij}$  is the active power which flows from node  $i$  to node  $j$ ;  $P_{ji}$  is the active power which flows from node  $j$  to node  $i$ ;  $P_n$  is the active power load of node  $n$ .

Due to the radiation of distribution network, the injected power of node  $n$  will only cause changes of the upstream branch power, does not affect the downstream branch power. As a result, the partial derivative of the end power of branch  $i - j$  is equal to the partial derivative of the head power of branch  $j - k$ . That is

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$$\frac{\partial P_{ji}}{\partial P_n} = \frac{\partial P_{jk}}{\partial P_n} \quad (5)$$

The partial derivative of the head branch power with respect to the end branch power is shown in formula (6):

$$\begin{bmatrix} \Delta P_{ij} \\ \Delta Q_{ij} \end{bmatrix} = S_{ij} \begin{bmatrix} \Delta P_{ji} \\ \Delta Q_{ji} \end{bmatrix} \quad (6)$$

$$S_{ij} = \begin{bmatrix} 1 + \frac{2R_{ij}}{U_j^2} P_{ji} & \frac{2R_{ij}}{U_j^2} Q_{ji} \\ \frac{2X_{ij}}{U_j^2} P_{ji} & 1 + \frac{2X_{ij}}{U_j^2} Q_{ji} \end{bmatrix} \quad (7)$$

The branch sensitivity to the injection power of a random downstream node is described by formula (8) and (9):

$$\begin{bmatrix} \Delta P_{ij} \\ \Delta Q_{ij} \end{bmatrix} = S_{in} \begin{bmatrix} \Delta P_n \\ \Delta Q_n \end{bmatrix} \quad (8)$$

$$S_{in} = S_{ij} \times S_{jk} \times \dots \times S_{mm} \quad (9)$$

According to the formulas above, the node injection power sensitivity for each node,  $S_{in}$ , to the first head branch power can be calculated. And the sensitivity of the power loss is get according to formula (3).

## ELECTRICITY DEMAND ELASTICITY MODEL

Under the real-time pricing environment, there will be some elasticity in electricity demand. According to the principles of economics, the relationship between the user demand  $D$  and the price  $\rho$  can be described by electricity demand curve<sup>[6]</sup>, which is shown in Figure 2.

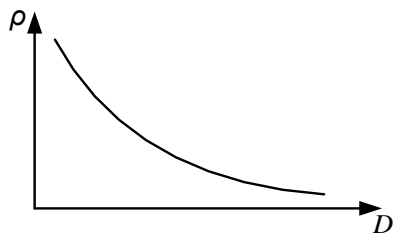


Fig.2 Power demand curve

Figure 2 illustrates that, if the price is high, the customers will make a timely response according to their own electricity consumption characters, their production mode and schedule will be adjusted to reduce electricity expenses. The customers can re-arrange the production in lower electricity price period so as to ensure normal producing. So the peak-valley difference could be reduced.

Because it's difficult to determine the electricity demand curve, it can be linearized in the range of small which

shown as follow:

$$\rho = d_1 + d_2 D \quad (10)$$

Where:  $\rho$  is the price;  $D$  is the demand;  $d_1$  is the intercept;  $d_2$  is the slope (negative).

The expression for demand elasticity  $\varepsilon_\rho$  is:

$$\varepsilon_\rho = (dD/d\rho)(\rho/D) = \rho/d_2 D \quad (11)$$

To estimate the impact of different types of load on certain distribution network, the current customer equipment capacity is taken as the maximum electricity demand  $D_{\max}$ , which is fixed in short term. And the demand of all types of customers should cross this maximum demand point, but the degrees of the price response for them are different, which means the demand elasticity of different types of customers differ from each other.

## REAL-TIME PRICING MODEL BASED ON NODAL LOAD SENSITIVITY

### Real-time pricing model of distribution network

The type of the load should be taken into account as the same as the load spatio-temporal distribution in the real-time pricing model of distribution network in. According to formula (8) and (9), the geographical distribution of loads and the density distribution of loads are reflected by the sensitivity of the distribution network nodal load. The type of the load will be illustrated by the demand elasticity. So real-time pricing model is presented as the fellows in this paper:

$$\rho_{i,t} = \omega \cdot \rho_{i,t,0} \cdot \left\{ \frac{S_{i,t}}{\text{avg}(S)} \right\}^{e^{\varepsilon_{\rho,i,t}}} \quad (12)$$

Where,  $\rho_{i,t}$  is the price for node  $i$  after the adjustment at time  $t$ ;  $\rho_{i,t,0}$  is the original price of node  $i$  at time  $t$ ;  $S_{i,t}$  is the sensitivity of node  $i$  at time  $t$ ;  $\text{avg}(S)$  is the average of all the node sensitivities during the calculation period;  $\varepsilon_{\rho,i,t}$  is the demand elasticity of node  $i$  under price  $\rho$  at time  $t$ ;  $\omega$  is the price adjustment factor. Price adjustment factor  $\omega$  can be adjusted properly according to the load type.

The calculation on the partial derivatives of the node price with respect to node sensitivity is shown as the following:

$$\frac{\partial \rho_{i,t}}{\partial S_{i,t}} = \frac{\omega \cdot \rho_{i,t,0} \cdot e^{\varepsilon_{\rho,i,t}}}{\text{avg}(S)} \left\{ \frac{S_{i,t}}{\text{avg}(S)} \right\}^{e^{\varepsilon_{\rho,i,t}} - 1} \quad (13)$$

At formula (13), the  $\text{avg}(S)$  is fixed as a constant during

the calculation period. The result of formula (13) is greater than 0 for  $\varepsilon_{\rho,i,t}$  is negative. That is, the larger the node sensitivity  $S_{i,t}$  is, the more expensive the node price will be.

The calculation on the partial derivatives of the node price with respect to demand elasticity factor is shown as the following formula:

$$\frac{\partial \rho_{i,t}}{\partial \varepsilon_{\rho,i,t}} = \omega \cdot \ln \frac{S_{i,t}}{\text{avg}(S)} \cdot \left\{ \frac{S_{i,t}}{\text{avg}(S)} \right\}^{e^{\rho_{i,t}}} \cdot e^{\rho_{i,t}} \quad (14)$$

In formula (14), when the sensitivity of node  $i$  at time  $t$  is greater than the average sensitivity of all the node,

$\frac{S_{i,t}}{\text{avg}(S)} > 1$ , and thus  $\ln \frac{S_{i,t}}{\text{avg}(S)} > 0$ , formula (12) is a

monotone increasing function. The increase in load will definitely cause a significant increase in the loss of the whole network. When the sensitivity of node  $i$  at time  $t$  is less than the average sensitivity of all the

node,  $\frac{S_{i,t}}{\text{avg}(S)} < 1$ , so that  $\ln \frac{S_{i,t}}{\text{avg}(S)} < 0$ , and formula

(12) will be a monotone decreasing function. The load increase will not cause an increase in proportion to the loss of the whole network.

According to (12), the real-time pricing method proposed in this paper reflects the location, size and demand elasticity of the load and other information. The location and size of the load mainly reflected in the node sensitivity, node with less sensitivity will reduce its real-time price in an appropriate way. On the contrary, node with high sensitivity will improve its real-time price. As to the node with high demand elasticity  $\varepsilon_{i,t}$ , the adjustment extent is relatively large, so that it can play the role of load curve peak load shifting better.

### Real-time price formed process

The first step is to get the parameter of the distribution network, load forecasting data of the next day and pricing information. The second step is power flow calculation. The method used in this paper is backward/forward sweep power flow calculation method which is based on two-dimensional deep coding. The third step is to calculate the sensitivity of each load node in the distribution network on the basis of the power flow calculation, and calculate the load level together with the demand elasticity  $\varepsilon_{\rho}$  for each node under the current price. At last, the real-time price for each node can be get using the real-time price model.

## CASE STUDY

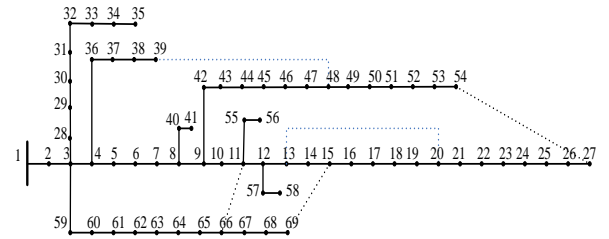


Fig.3 IEEE69-nodes distribution system

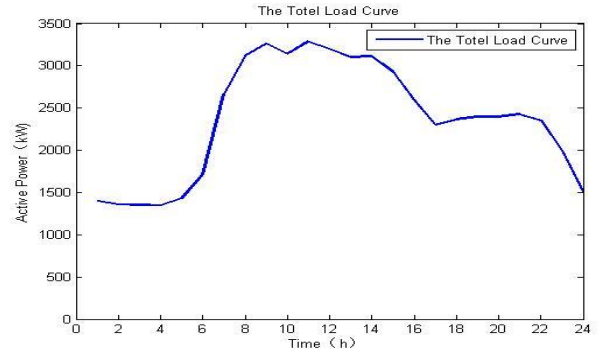


Fig.4 Total load curve of 69 nodes distribution system

The example used in this paper is IEEE69-nodes distribution network, the network structure is shown as Figure 3. This system contains industrial, commercial and residential three types of loads. Each node takes the original loads as the maximum load value. According to the typical daily load curve in reference [7], the daily load fitting-curve for each node in the IEEE69 system can be get. Its total load curve is shown in Figure 4.

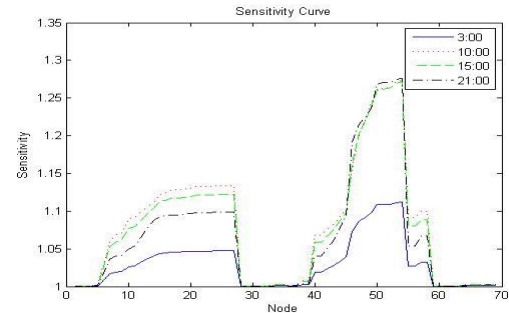


Fig.5 Sensitivity of system's nodes at different times

The sensitivities for different nodes at each time is shown in Figure 5. The sensitivities differ greatly from each other. The highest load sensitivity is in Node 54, for the branch which it stands in takes the heaviest load. On the same branch, the farther the distance between the node and the feeder outlet, the higher the sensitivity of the node is. This can reflect the geographical distribution of loads efficiently. Meanwhile, the sensitivity can reflect the load size and network structure to some extent.

The size of sensitivity for each node per day goes the same as the actual load curve trend, this happens to fit the real-time pricing principles. According to the two points above, it is reasonable to take the node sensitivity as the basis to real-time price.

If the residential, commercial, industrial actual maximum

load art taken as 0.90, 0.85, 0.80 of their equipment capacity, and the customer will take the equipment capacity as the maximum consumption when the price is 0, then the linearized load elasticity fitting curve will be figured out according to formula (10). The demand elasticity of different times and different types can be figured out by formula (11). Electricity price adjustment factor  $\omega$  is set to be 1.1. The electricity price can refer to the Beijing electricity price, which is shown in Table 1, and then the real-time price will be figured out by the model proposed in this paper. Figure 6 gives out the information about real-time price of node 10, node 35, node 53 and node 66.

Table 1 Information of each type load price

Load Type	Industrial	Commercial	Residential
Price(¥/kWh)	0.978	1.194	0.488

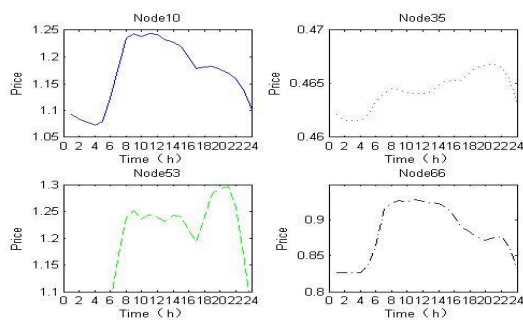


Fig.6 Real-time electricity price of each node

In Figure 6, node 10 is commercial load, node 35 is residential load, node 53 and node 66 are industrial loads.

As be seen from the figure, the method established in this paper can reflect the impacts of different types of the loads, the location and the size of the load. The real-time pricing curve per day agrees with the typical load curve for residential load node 35. At the peak of the total feeder load curve, the electricity price will be increased correspondingly. At the same time, as it stands near the feeder, the sensitivity is rather small, so the real-time price per day will be reduced to some extent. This will benefit the economical operation of distribution network. For the industrial load nodes 53 and 66, the difference of electricity price between them is quite large, that is because the load of the branch which node 66 stands in is rather small. The impact of the load variation on the economical operation of the whole network is small, so the overall price level is low. However, the trend of the variation for the overall price level is the trend of the load variation per day, which reflects that this pricing method can efficiently realize the peak load shifting. The branch load where the node 53 stands in is large, the impact of the load variation on the economical operation of the whole network is notable, so the overall price level should be improved to some extent. The trend of its variation reflects the trend of the load variation per day as well. Based on the analysis above, the pricing method can reflect the geographical distribution and the character

of the loads.

## CONCLUSIONS

Based on the simplified distribution network, the formula of the loss of distribution network sensitivity to the nodal load is deduced according to the distribution structure characters in this paper.

The real-time electricity price for distribution network reflect the impact of the consumption character by the demand elasticity, and reflect the impact of the load distribution by the nodal load sensitivity to the network power losses. The distribution network real-time pricing method proposed in this paper can combine those two characters effectively. It can reflect the normal operation level of the distribution network, that is, the price near the feeder is low, and the price of the branch with heavy load is high. In the case of customer participation and response, the operation economic of distribution network can be effectively improved.

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