

# DISTRIBUTED OPTIMISATION WITH RESTRICTED EXCHANGS OF INFORMATION: CHARGING OF AN ELECTRIC VEHICLE FLEET

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# ABSTRACT

The multiplication of flexible loads into electrical power systems allows to develop distributed optimisation techniques. The goal is to achieve the best compromise between the individual objectives of each load and the general interest which could be for instance the safety of the grid or the efficiency of the power generation. However the algorithms of distributed optimisation mostly rely on an iterated exchange of information, such as the dual variable and the decision of each agent. This exchange therefore supposes a bidirectional communication which cannot be considered as granted in operational contexts. This paper investigates the utilisation of the Alternating Direction Method of Multipliers under restricted exchanges of information. The case study consists in an electric vehicle fleet which has to comply with a maximum power limit. It is established that a close to optimal behaviour can be achieved using frequent broadcast messages instead of duplex communication.

Keywords: Electric vehicle fleet, Alternating Direction Method of Multipliers, Stochastic Dynamic Programming.

# **INTRODUCTION**

The diversification of the electrical devices pave the way to a potential increase of the electricity demand, in particular because of the growth of electric vehicles. This increase of the consumption compels to develop a sustainable power generation so as to ensure the relevance of electric mobility from well to wheel. In addition channelling this electricity down to vehicle charging stations may bring a significant overload on the grid. The sizing of the transport and distribution networks have not been done accordingly. In particular the charging of numerous vehicles in residential areas could make the peak consumption unbearable for both production units and distribution systems.

However new electricity consumers such as electric vehicle offer some levels of flexibility. Indeed their consumption consists in the charging of a battery which can be shifted in time. This shift has no consequence on the service provided to the user, assuming he announced a departure time when he expects his vehicle to be charged. This characteristic is shared with other flexible consumptions such as heating, but electric vehicles may also be temporary discharged. This discharge could relieve stress on generation units and transport lines. Thus some situations could avoid to strengthen their infrastructures or to delay this adaptation [1]. However it will also bring a greater ageing of the battery in comparison with a charging only focused on the minimisation of the battery losses and ageing. It is therefore mandatory to evaluate this additional decay and to compare it to the service provided to the grid.

As the impact of electric vehicles on a grid cannot be significant but if many act jointly, many studies have tackled the distributed control of electric vehicle fleets [2-3]. However the distributed optimisation methods widely assume a context where communication is possible, fast and free of charge, between vehicles and/or between vehicles and a central agent. Then an iterative resolution of the problem can be carried out and the solution is enforced once the algorithm converged.

However in the operational context of an electric grid, this free and fast communication becomes a very strong hypothesis. It would mean to build a communication network alongside the power network, or to use existing communication infrastructures with inherent risks of delay and loss of information.

The scope of this study is to investigate the distributed optimisation of the charging of an electric vehicle fleet. The resolution of this problem is first presented to build a reference case under the assumption of a bidirectional communication between each vehicle and a central agent. Then only broadcast signals are supposed to be possible. Such signal could be for instance send through power line communication within a distribution district which is supplied by a single substation. Next section will introduce the optimal charging problem and the considered case study. The resolution algorithm will also be presented, in a general case and in the restricted communication context. Then the third section will present the resolution of the problem that each vehicle has to solve, as required by distributed optimisation frameworks. The last section will finally present and discuss the results.

### OPTIMAL CHARGING PROBLEM UNDER A MAXIMUM POWER CONSTRAINT

### **Problem Presentation**

The case study that is here considered is a distribution district. The substation which supplies the district has a rated power  $P_{rated}$ . A non-flexible component of the consumption  $P_{conso}(t)$  is introduced and fluctuates along



the day. The remaining available power is then:

$$P^{\#}(t) = P_{rated} - P_{conso}(t)$$

Within this district, electric vehicles are introduced with a flexible consumption and the possibility to be discharged. The goal of the district power management is therefore to respect the power limit while providing the best charging power for the vehicles. That is to say that the battery should be as charged as possible by the departure time of the vehicle. This objective function of each vehicle is named  $f_i(P_i)$  with  $P_i$  the vehicle charging power. The future has to be anticipated because of the temporal coupling introduced by the batteries. Moreover the problem is stochastic as the vehicles are randomly moving. Within this case study, the non-flexible consumption is assumed to be perfectly forecasted and therefore deterministic. This assumption could be cleared by using a stochastic modeling, similar to the one used for the vehicles.

The optimal management problem is the minimization of the expected total cost over a time horizon:

 $\min_{\substack{P_i^{t_0}\\P_i^{t}}} E\left(\sum_{t=t_0}^T \sum_{n=1}^N f_i(P_i^t)\right)$  such that for all t

$$\sum_{i=1}^{N} P_i^t \le P^{\#}(t)$$

#### **Resolution without information constraint**

The previous problem is a sharing problem which could be written as:

$$\min_{\substack{x,z\\s.t.}} f(x) + g(z)$$

where g is the indicator function of the set  $C = \{\sum z_i \le P^{\#}\}$ . Such a problem can then be solved in an efficient way using the Alternating Direction Method of Multipliers ADMM [4].

$$x_i^{k+1} := \operatorname{argmin}_{r_i} f_i(x_i) + \frac{\rho}{2} \|x_i - z_i^k + u_i^k\|^2$$
$$z^k := \prod_{c} (x^{k+1} + u^k)$$
$$u^{k+1} := u^k + x^{k+1} - z^{k+1}$$

Along these iterations, each vehicle first decides its charging power to minimise its own individual cost plus a coordination term. As soon as all the vehicles decisions are gathered, this decision is projected on the feasible set C. Finally the equality constraint between x and z is enforced by updating the scaled dual variable u with a gradient descent.

#### **Resolution with information constraint**

On the basis of the previous resolution, the accessible information are reduced to two signals. The **total consumption of the district** can be measured at the substation level and it is therefore supposed to be accessible at any time. The **dual variable value** is the second signal that is necessary in the proposed resolution. As soon as the total consumption power is measured, it is possible to project it on the feasible set and to update the dual variable accordingly. Then this scalar value needs to be broadcasted to all the vehicles. This resolution framework meets with operational contexts where messages are sent to all agents at the same time [5]. The electrical grid itself can then be used to periodically send a value for instance by power line communication or broadband over powerline. With this limited exchange of information, these two values meet the operational feasibility and a sufficient coordination to carry out a resolution fulfilling the power constraint - although the performance cannot be as good as in the unrestricted case. The proposed approach consists in substituting the iterations of the optimisation algorithm by real power exchanges with frequent broadcast of the dual variable. First each vehicle decides its optimal charging power on the basis of its own objectives and of the previous dual variable value. The optimal power is applied right away. At the substation level the total power of the district is measured and compared to the rated power. There is no guarantee that the rated power is always fulfilled. If not, a new value for the dual variable is computed and broadcasted. This resolution is similar to the iterations of ADMM algorithm except that real power exchange immediately enforced are used instead of iterated exchanges of information until convergence. The time step must then be much finer than in the unrestricted resolution.

### OPTIMAL STRATEGY FOR A VEHICLE CHARGING POWER

The distributed resolution of the fleet charging requires that each vehicle is able to solve the problem of its own optimal charging taking into account the term coming from the coordination at the fleet scale:

$$\min_{P_i} E\left(\sum_{t=t_0}^T f_i(P_i^t) + \frac{\rho}{2} \|P_i^t - z_i^{k,t} + u_i^{k,t}\|^2\right)$$

The resolution of this problem has to be performed very often as at each time step, each vehicle must solve it at each iteration. This calls for a resolution using Stochastic Dynamic Programming SDP [6]. The optimal strategy which will then be computed will contain the best decision for the charging strategy for all configuration of the state vector  $x_i = (E^{\#}, SoC, \tau, \lambda)$ , where  $E^{\#}$  is the capacity of the vehicle battery, SoC is its state of charge,  $\tau$  is the remaining duration before the vehicle departure, gathers the  $z_i^k - u_i^k$  quantity. However, such a λ minimisation means to anticipate the future variations of the own vehicle - which is possible if the departure time is supposed to be announced by the driver – but also the future values of the dual variable *u* and of the variable *z*. These quantities depend on the future state of the fleet which cannot be known as vehicles travels are stochastic. Therefore the evolution of the  $\lambda$  value is considered as a stochastic process which is modelled by an autoregressive model [7].

The objective function  $f_i$  takes into account battery losses,



battery ageing and the vehicle mobility:

$$f_i(P_i) := \sum_{t=t_0}^{t} \left( f_{loss}(t) + f_{age}(t) + 1_{t=t_{dep}^i} f_{mob}(t) \right)$$

where  $f_{loss}$  and  $f_{age}$  respectively compute the losses and the ageing of the battery,  $f_{mob}$  stands for the user mobility cost and penalize any missing energy in the battery at the departure time  $t_{dep}$ . They are computed with the following expressions:

$$P_{loss} = \alpha_{loss} P(t)^{2}$$
  

$$d_{i} = \alpha_{age} DoD^{\beta_{age}}$$
  

$$C_{mob} = \alpha_{mob} (1 - SoC)^{2}$$

A quadratic model is used for the losses. The battery ageing is assessed using the depth of discharge of the current cycle DoD and the mobility cost is a quadratic function depending on the state of charge. The resolution by SDP consists in the resolution of the Bellman equation:

$$J_{\tau=0}^{*}(x) = f_{mob}(SoC)$$

$$J_{\tau}^{*}(x_{\tau}) = \min_{P_{i}} f_{loss}(P_{i}) + f_{age}(P_{i}) + \frac{\rho}{2} \left\| P_{i} - \lambda \right\|^{2}$$

$$+ E \left( J_{\tau-\Delta T}^{*} \left( f_{dyn}(x_{\tau}, P_{i}) \right) \right)$$

The result is then a four dimension matrix – as the state space has four dimensions – which describes the best decision on a discretised grid of the state space. Figure 1 presents cross sectional views of this strategy depending on the battery state of charge and of the remaining time before departure, for different values of  $\lambda$  and a fixed battery capacity  $E^{\#} = 85kWh$ . Iso-power curves are also plotted. The first panel – left-top – stands for the situation when the fleet interest needs the vehicle to discharge at full power, no matter what is its individual situation. Such a behavior is achieved using extremely negative values of  $\lambda$  so that the coordination term prevails against its individual objectives. Symmetrically the last panel –



-100kW -80kW -60kW -40kW -20kW 0 20kW 40kW 60kW 80kW 100kW Figure 1: Cross sectional views of the optimal strategy for the charging power

right bottom – is the case where the vehicle must consume a power as huge as possible, in the limit of its rated power and current state of charge. The two intermediate panels represent situations where the coordination term and the individual costs of the vehicle have a similar magnitude. The vehicle charging power is then modulated depending on the state of charge and the remaining time before departure: a vehicle with a nearly empty battery which must leave soon will have a greater charging power than a vehicle with a nearly full battery which has a long time before departure. The coordination term still brings a modulation of the charging strategy so as to find a compromise between the individual costs and the global constraint.

#### **RESULTS AND DISCUSSION**

#### Non flexible consumption profile

The case study is built on a distribution district whose substation has a rated power of  $P_{rated} = 100kW$ . The temporal profile of  $P_{conso}$  is generated from historical data of electricity consumption in France, freely available from French TSO RTE [8]. These consumption profiles is aggregated at a country scale and gather domestic, industry and services consumption, which are very unlikely to be observed within the same distribution district. Nevertheless it provides a relevant context to assess the adaptation capacity of the proposed resolution.

#### Vehicle availability scenario

The mobility scenario of each vehicle within the fleet is randomly drawn from probability densities. For the battery capacity, it is built on the market shares of electric vehicles in 2014. The distributions of arrival and departure hours match a fleet which arrives in the morning and leaves in the evening [9]. Finally the probability of the initial state of charge of the battery is deduced from the length of the last trip. These different values are only set from a statistical perspective and a 200 vehicle fleet is simulated over a year. When a vehicle is plugged to a charging station, its state vector is supposed to be known.

#### **Temporal trajectories**

Figure 2 presents temporal trajectories of the charging power of the fleet during a week. On the top panel, the maximum power that the vehicles can consume is in red. In blue is the total power that is actually consumed. The evolution of the dual variable is presented on the middle panel. This dual variable can be seen as the energy price. It is equal to zero when the constraint is not activated. Its value stands for the restriction that the vehicles undergo on their individual objectives so as to fulfil the global constraint. Because of the chosen mobility scenario, this pressure reaches its maximum during the morning. Finally the last panel shows the individual charging trajectories of each vehicle. It can be noticed that some vehicles can be temporary discharged for the benefit of others to make the most of the fleet interest.







Figure 2: Trajectories of a fleet charging under a maximum power constraint

#### **Resolution with information constraint**

When information exchanges are limited, the maximum power constraint cannot be fulfilled at all time. Figure 3 presents the trajectories that can be observed during a winter week and a summer week. In winter vehicle have less available power because of the greater non flexible consumption. The constraint is then more difficult to respect. Although the constraint is regularly violated, it can be noticed that the fleet reacts quickly to these violations and that the general pattern is respected. Moreover the violation amplitude can be significantly lowered by increasing the frequency of the dual variable update as presented in Table 1. The frequency of the violation cannot be set to zero because the possibility of a violation is not anticipated in the presented resolution. But the impact on the mean amplitude is very significant.

Updates	Violation	Mean violation
frequency	frequency	amplitude
10min	34%	9kW
5min	20%	5kW
1min	12%	1.5kW
30s	10%	0.7kW
10s	9%	0.3kW
58	8%	0.1kW

Table 1: Constraint violation depending on the frequency of the dual variable update



*Figure 3: Trajectories of a fleet charging when communication is restricted* 

#### CONCLUSION

This study investigated the operational feasibility of a distributed optimisation algorithm for the control of an electric vehicle fleet. First the optimal charging of this fleet has been presented in order to meet a maximum power constraint. This distributed resolution was based on the Alternating Direction Method of Multipliers and Stochastic Dynamic Programming. As in many others distributed scheme, a bidirectional communication between each vehicle and a central agent is necessary. As such a communication may not be possible in a real application, the proposed resolution is then adapted using only two signals. The first one is the measure of the total power - which is supposed to be accessible in the district substation. The second one is a broadcast dual variable comparable to an energy price - which is sent to all vehicles for instance through power line communication. It is shown that the constraint shape is then roughly respected, although some violations may occur. The amplitude of these constraint violations can be put as low as required thanks to a more frequent update of the dual variable. This preliminary results are very hopeful for an operational distributed control of flexible loads.

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