MODELLING OF THREE-PHASE FOUR-WIRE LOW-VOLTAGE CABLES TAKING INTO ACCOUNT THE NEUTRAL CONNECTION TO THE EARTH

Frédéric OLIVIER
University of Liège – Belgium
frederic.olivier@uliege.be

Raphaël FONTENEAU
University of Liège – Belgium
raphael.fonteneau@uliege.be

Damien ERNST
University of Liège – Belgium
dernst@uliege.be

ABSTRACT

Local energy communities (LEC) usually take place at the level of the low-voltage distribution networks, which are inherently unbalanced, due to single-phase household appliances and distributed generation. To simulate and optimise the behaviour of a LEC, the three phases and neutral must be modelled explicitly. This paper aims at assessing numerically the influence of the modelling of the earth and the connection between the neutral and the earth, in terms of voltages and currents. The simulations are performed on a real Belgian low-voltage feeder supplying 19 houses, which are all equipped with a smart meter measuring the mean voltage, current, active and reactive power every minute for each phase. The simulations show that the difference introduced by the explicit modelling of the earth using Carson's equations is much smaller than the difference between the simulations and the real measurements.

INTRODUCTION

Local energy communities (LEC) usually take place at the level of the low-voltage distribution networks, which are inherently unbalanced, due to single-phase household appliances and distributed generation. The common assumption made in high voltage and medium voltage networks, that the three phases are balanced, can no longer be made. To simulate and optimise the behaviour of a LEC, the three phases and neutral must be modelled explicitly. Indeed, in some cases, the current in the neutral is of the same order of magnitude as the phase ones, which can have two effects: on the one hand, if the neutral conductor is not grounded, the neutral current creates a neutral voltage, which modifies the voltage of the phases. This phenomenon is called neutral point shifting [1]. On the other hand, if the neutral conductor is grounded, there is an unchecked current flowing through the earth. In either case, this should be taken into account.

This paper aims at reviewing the main assumptions made to model three-phase electrical line, and more specifically at assessing numerically the influence of the modelling of the earth. This is done in the setting of a real Belgian low-voltage (LV) distribution feeder. Each house has been equipped with a smart meter measuring the one-minute average voltage, current, active and reactive power for each phase. The minute has been chosen to clearly exhibit the imbalance in the network and the variability of the loads. A longer averaging period would smooth out those characteristics [2], which bring out the impact of the neutral and earth modelling.

The paper is organized as follows. The first section will review the common modelling hypotheses and simplifications for LV electrical lines. The second section will assess numerically the impact of Carson's equations to model the earth return path, and finally, the third section will compare the simulation results with the actual measurements from the smart meters.

ELECTRICAL LINE MODELLING

In this study, the electrical lines are composed of four conductors (three phases and one neutral). In the simulations, they are modelled as PI equivalents with an impedance matrix, which corresponds to the resistance, self- and mutual impedance of the conductors, and a shunt admittance matrix, representing the capacitance between the conductors. This admittance matrix is divided in two halves, one at the sending end of the line and the other at the receiving end. Both matrices are computed using PowerFactory by entering the physical constants of the conductors (material, section, etc.). This implies that the impedance does not depend on the length of the cable, i.e. that the leakage flux at the end of the lines can be neglected [3]. Finally, since the aim of this study is to model LEC to simulate their operations and optimize their behaviour, only power flows in steady-state will be considered (no harmonics or fault studies).

Carson's equations and earth return path

PowerFactory uses Carson's equations [4] to update the impedance matrix to take into consideration the earth return path. Let Z_4 be the 4x4 impedance matrix with only the resistance of the conductors and their self- and mutual impedance. Carson's equations provide a way to modify Z_4 to take into account the earth return path [5]. Let $\hat{Z}_4$ be the updated version of Z_4. Thanks to the work of Kersting [6], the earth impedance can be extracted from $\hat{Z}_4$ to create Z_5, a 5x5 where the earth return path is modelled explicitly as an extra-conductor. This formulation has been used to solve power flow in [7].
However, Carson's equations have been originally designed for overhead lines with widely spaced conductors, which is not the case in LV networks. Moreover, even if it has been shown that they can be applied to underground cables for the mains frequency [8], they use image theory to compute the earth return path, and thus assume the latter is parallel to the conductors, an hypothesis which may not hold in LV distribution system presenting a complex topology and a proximity to underground pipes, which can provide a path with a smaller impedance.

Kron's reduction

By assuming that all buses have a neutral conductor perfectly connected to the ground, one can reduce the size of the impedance matrix from 4 to 3. Indeed, the neutral voltage difference at the beginning and the end of the line is zero. As explained in [9],

$$\Delta V = Z_4 I$$

$$\begin{pmatrix} \Delta V_{abc} \\ \Delta V_n \end{pmatrix} = \begin{pmatrix} Z_{abc} & Z'_{abcn} \\ Z_{abcn} & Z_n \end{pmatrix} \begin{pmatrix} I_{abc} \\ I_n \end{pmatrix}$$

$$\Delta V_n = 0$$

$$\Delta V_{abc} = (Z_{abc} - Z'_{abcn} Z_n^{-1} Z_{abcn}) I_{abc} = Z_3 I_{abc}$$

where ΔV represents the voltage differences along the line for the four conductors and I is the current trough them. With Kron's reduction, the impedance matrix is split into four submatrices, one related to the phases Z_{abc}, another related to the neutral Z_n, and the last ones linking the two, Z_{abcn}.

Kron's reduction can be applied to both Z_4 and $\hat{Z}_4$, depending on the use of Carson's equations. If all nodes are grounded, Kron's reduction does not modify the simulation results, however care must be taken if this reduction is used for network with ungrounded neutral, as it will lead to erroneous neutral voltages.

Sequence impedance matrix

Instead of using phase impedance matrices to represent the lines, one could use sequence impedance matrices. If the lines are considered to be transposed regularly, the sequence impedance matrix will be diagonal. However, zero sequence impedance is commonly not given by manufacturers as it depends on the network earthing. The positive sequence impedance is commonly multiplied by a factor between 2.5 and 3.5 for lines without ground return [3]. However, [10], [11] have shown that approximating the zero sequence impedance with the positive one can lead to large errors in the voltages, justifying the use of phase impedances.

Shunt admittance matrix

Finally, given that the shunt capacitances of LV networks are small, even for underground cables. As in [7], [12], [13], the admittance matrices will be neglected.

TEST CASES

Four network configurations have been selected to analyse the influence of the earth modelling.
1. No buses are grounded, except the neutral point of the distribution transformer which is perfectly grounded.
2. The neutral of specific buses is grounded through a 0.5-Ω impedance.
3. The neutral of specific buses is perfectly grounded.
4. The neutral of all buses is perfectly grounded.

TEST SYSTEM AND MEASUREMENT CAMPAIGN

The test network used for this study is an existing Belgian LV distribution network with a star configuration 400V/230V. It is composed of one feeder made with underground cables of the type EVAVB-F2 3x95 + 1x50. Detailed unbalanced three-phase four-wire modelling of the network has been used according to [7] based on the data provided by the Distribution System Operator (DSO) (topology, line length, cable type, etc.). The actual configuration of the network is the first one (ungrounded neutral). In configurations 2 and 3, the neutral is assumed to be grounded at the buses marked with a red square in Figure 1.
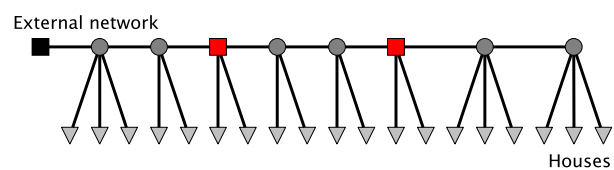

Figure 1 – Graphic representation of the test network.

The network supplies 19 houses, all equipped with a three-phase smart meter. Each smart meter provides phase-to-neutral voltage, current, active and reactive power measurements for each phase, as a one-minute average. The beginning of the feeder is also equipped with a smart meter. It is important to note that the phase identification of all smart meters is known thanks to the algorithm presented in [14].

RESULTS

Simulations have been performed on the test network for each network configurations, on the one hand without using Carson's equations (i.e. using Z_4) and on the other hand, using them to represent the earth return path (i.e. using Z_5). A Newton-Raphson algorithm has been applied to solve the power flow equations expressed in rectangular form. The consumption and production profile of the different houses are the ones form the smart meters, thus reflecting the true operations and imbalance of the network.

The difference between the simulation results from the two earth modelling techniques are assessed with two measures:
1. The mean absolute difference (MeanAD):

$$\frac{1}{n} \sum_{i=1}^{n} \frac{|V_i - V_i'|}{230}$$

2. The maximum absolute difference (MaxAD):

$$\max_i \frac{|V_i - V_i'|}{230}$$

where V_i and V_i' are two voltage time series, and n is the their length. All results are represented with box plots,

where the samples are the measures of all time series. For example, since there are 19 three-phase buses, the boxplot will represent the distribution of the mean absolute error for the 84 voltage time series. On each box, the central mark is the median, while the edges of the box are the 25th and 75th percentiles. Outliers are identified and plotted individually, while the whiskers extend to the most extreme data points not considered as outliers.

Modelling the earth return path

Phase to neutral voltages

As can be seen in Figure 1, the median of the MeanAD is small, approximately $4 \cdot 10^{-4}$ p. u. As expected, the MeanAD is equal to 0 when the neutral is not grounded. Indeed, no currents flow through the earth, and since the sum of the currents in the four conductors is equal to zero, the earth has no influence on the simulation results. The MeanAD is gradually larger in configuration 2 where specific buses are grounded through an impedance, 3 where specific buses are perfectly grounded, and 4 where all buses are perfectly grounded, the earth's influence being most prominent in that case.
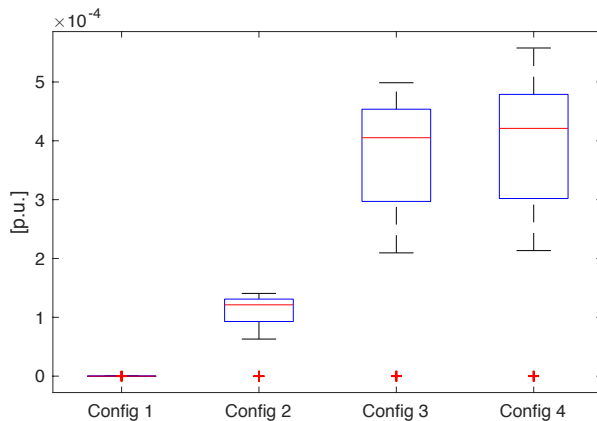

Figure 2 – Boxplot of the mean absolute differences between the phase-to-neutral voltages when the impedance of the earth is not modelled, and when Carson's equations are used.

The same tendencies can be observed for the MaxAD, with median of $2.5 \cdot 10^{-3}$ in configuration 4, as shown in Figure 3.
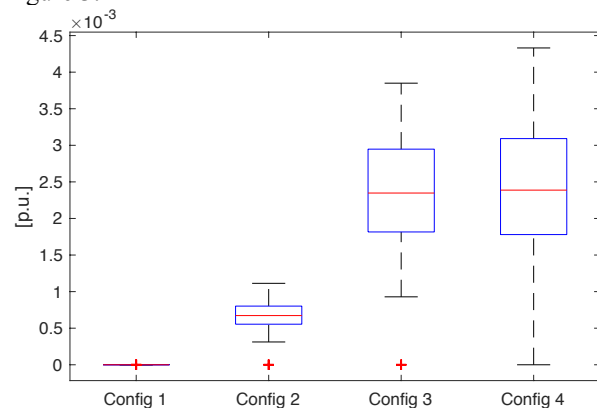

Figure 3 – Boxplot of the maximum absolute differences between the phase-to-neutral voltages when the impedance of the earth is not modelled, and when Carson's equations are used.

Currents

In the case of phase currents, the MeanAD is taken relatively to the current intensity at the same time, i.e.

$$\frac{1}{n}\sum_{i=1}^{n}\frac{|I_i - I_i'|}{I_i}$$

where I is the magnitude of the current.

As Figure 4 shows, the differences between the phase currents when Carson's equations are used is negligible, approximately a difference of 0.05%.
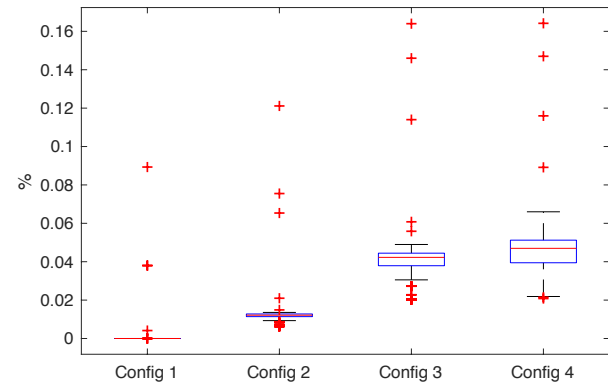

Figure 4 – Boxplot of the mean absolute difference between the currents in the phase conductors. In this case, the MeanAD is taken relatively to the current magnitude.

Kron's reduction

Kron's reduction can only be used in configuration 4, without leading to simulation errors. In that case, simulations using Z_3 and Z_4 are completely equivalent. The same can be said for simulations using $\hat{Z}_3$, $\hat{Z}_4$ and Z_5. The only difference is the computation burden as the size of the network matrix is uselessly larger.

Conclusions

Given the uncertainty in the applicability of Carson's hypotheses to LV networks, the increased computation burden to use 5x5 matrices to represent the lines, and the small value of the MeanAD and MaxAD, we advise to use 4x4 matrices, and to consider the earth as a single electrical point, i.e. the only ground of the electrical equivalent circuit.

Comparison between simulations and measurements

Taking into considerations the conclusions from the previous section, Figure 5 and Figure 6 show the error between the phase-to-neutral voltage measurements and the voltages simulated in the exact same conditions (i.e. same network topology, same cables, same consumption and production in the houses), when the earth is not modelled with Carson's equations. It is expected that configuration 1 leads to the smallest error, as the neutral of the test network is ungrounded. With a median of approximately 0.001 p.u., Figure 5 highlights a reasonable error, indicating that the modelling of the test LV network was done properly. Compared to the

previous sections, the MeanAD and MaxAD are approximately 10 times larger, further showing that the impact of the earth modelling is small, and can be neglected in front of the errors between simulations and measurements. These errors can be explained by several reasons: (i) the network topology in the Geographical Information System (GIS) of the DSO does not reflect reality. (ii) There is noise is the measurements and perhaps unmonitored consumption. (iii) The line impedances are calculated theoretically and not measured on site, etc.
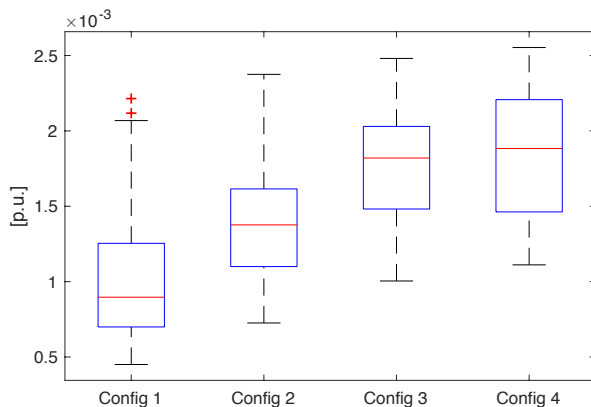

Figure 5 – Boxplot of the mean absolute errors between the phase-to-neutral voltages that are simulated and measured.
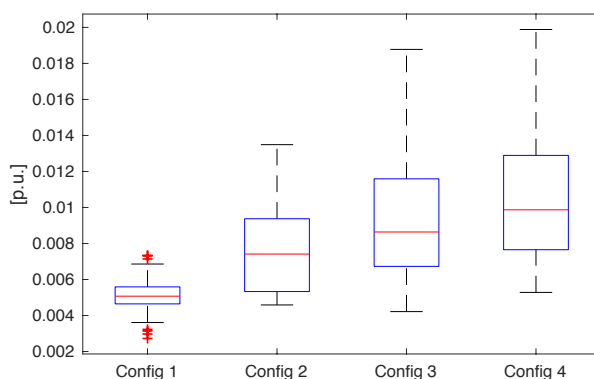

Figure 6 – Boxplot of the maximum absolute errors between the phase-to-neutral voltages that are simulated and measured.

CONCLUSION

This paper provides guidelines to help modelling LV electrical lines. The first one is that Kron's reduction should only be used when all the buses of the network are perfectly grounded, a case which is rarely met LV networks. Second, Carson's equations should be used with caution as their hypotheses may not hold in LV networks. For the sake of simplicity, we advise to consider the earth as a single electrical point in the network, to which all the grounded neutral points are connected, with or without a grounding impedance. Indeed, the voltage differences brought by Carson's equations are much smaller than the global error between simulations and measurements.

REFERENCES

[1]   L. Degroote, B. Renders, B. Meersman, and L. Vandevelde, "Neutral-point shifting and voltage unbalance due to single-phase DG units in low voltage distribution networks," in *Proc. of 2009 IEEE Bucharest PowerTech*, 2009.

[2]   A. Wright and S. Firth, "The nature of domestic electricity-loads and effects of time averaging on statistics and on-site generation calculations," *Appl. Energy*, vol. 84, no. 4, pp. 389–403, 2007.

[3]   A. J. Urquhart, "Accuracy of Low Voltage Distribution Network Modelling," Loughborough University, 2016.

[4]   J. R. Carson, "Wave Propagation in Overhead Wires with Ground Return," *Bell Syst. Tech. J.*, vol. 5, no. 4, pp. 539–554, Oct. 1926.

[5]   W. H. Kersting and R. K. Green, "The application of Carson's equation to the steady-state analysis of distribution feeders," in *Proc. of 2011 PES Power Systems Conference and Exposition*, 2011.

[6]   W. H. Kersting, "The computation of neutral and dirt currents and power losses," in *Proc. of 2004 PES Power Systems Conference and Exposition*, 2004.

[7]   R. M. Ciric, A. P. Feltrin, and L. F. Ochoa, "Power flow in four-wire distribution networks-general approach," *IEEE Trans. Power Syst.*, vol. 18, no. 4, pp. 1283–1290, Nov. 2003.

[8]   N. Srivallipuranandan, "Series Impedance and Shunt Admittance of an Underground Cable System," University of British Columbia, 1986.

[9]   W. H. Kersting, *Distribution System Modeling and Analysis*. Boca Raton, 2006.

[10]   D. F. Frame, G. W. Ault, and S. Huang, "The uncertainties of probabilistic LV network analysis," in *Proc. of 2012 IEEE Power and Energy Society General Meeting*, 2012.

[11]   W. H. Kersting and W. H. Phillips, "Distribution feeder line models," *IEEE Trans. Ind. Appl.*, vol. 31, no. 4, pp. 715–720, 1995.

[12]   D. Das, H. S. Nagi, and D. P. Kothari, "Novel method for solving radial distribution networks," *IEE Proc. - Gener. Transm. Distrib.*, vol. 141, no. 4, p. 291, Jul. 1994.

[13]   K. M. Sunderland and M. F. Conlon, "4-Wire load flow analysis of a representative urban network incoparing SSEG," in *Proc. of 2012 47th International Universities Power Engineering Conference (UPEC)*, 2012.

[14]   F. Olivier, A. Sutera, P. Geurts, R. Fonteneau, and D. Ernst, "Phase Identification of Smart Meters by Clustering Voltage Measurements," in *Proc. of PSCC 2018*, 2018.